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Military Operations Research

Spring 1996

V2, N1

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MORS
MILITARY OPERATIONS RESEARCH SOCIETY

Military Operations Research

A publication of the Military Operations Research Society

The Military Operations Research Society is a professional society incorporated under the laws of Virginia. The Society conducts a classified symposium and several other meetings annually. It publishes proceedings, monographs, a quarterly bulletin, *PHALANX*, and a quarterly journal, *Military Operations Research*, for professional exchange and peer criticism among students, theoreticians, practitioners and users of military operations research. The Society does not make or advocate official policy nor does it attempt to influence the formation of policy. Matters discussed or statements made in the course of MORS symposia or printed in its publications represent the opinions of the authors and not the Society.

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Note from the Publisher:
This is the first issue of *Military Operations Research* published under the auspices of our new Editor Dr. Gregory S. Parnell. Greg is a professor at Virginia Commonwealth University and a long-time participant in MORS' activities. Please note that there are several changes in this new volume. First, we have a new Editorial Policy and instructions on Submission of Papers (see pages 2-3). And we have added Executive Summaries of each Journal article at the beginning of the issue.

We hope you will find these changes useful. As always, the staff and Board welcome your comments and suggestions.

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CALL FOR PAPERS

SPECIAL ISSUE ON

MILITARY OPERATIONS RESEARCH METHODS

FOR

FUTURE R&D CONCEPT EVALUATION

Limited research and development (R&D) budgets make it imperative that the United States analyze the potential operational benefits of future system concepts and select the most promising concepts for further R&D spending. Military operations research offers several techniques to help senior DoD decision-makers prioritize future R&D concepts. The purpose of this special issue is to describe the most effective techniques in use and to propose improvements to military R&D concept evaluation techniques.

Since many of the DoD R&D concept evaluation studies are classified, we are willing to publish the unclassified techniques or the techniques with notional data. However, in accordance with our editorial policy, we require certification from a senior decision-maker that the military R&D concept evaluation techniques were used.

Interested authors should submit **abstracts** by **November 30, 1996** and **papers** by **January 31, 1997**. The papers should be submitted in accordance with our current editorial policy. All papers will be refereed.

We are also seeking volunteers to serve as guest editors, associate editors, and referees for this special issue.

Please contact me if you are interested in authoring a paper or serving as an editor/referee for this special issue.

DR. GREGORY S. PARNELL
Editor, *Military Operations Research*
Department of Mathematical Sciences
Virginia Commonwealth University
Oliver Hall, 1015 W. Main Street
P.O. Box 842014
Richmond, VA 23284-2014
Phone: 804-828-1301, ext. 133/Fax: 804-828-8785
Email: gparnell@vcu.edu and gsparnell@aol.com

Editorial Policy and Submission of Papers

Military Operations Research: What's Changed and What Hasn't?
Gregory S. Parnell, Editor, *Military Operations Research*

What hasn't changed?

We want to maintain and improve upon the high quality of articles published in *Military Operations Research.* The first Editor, Dr. **Peter Purdue**, and his Associate Editors have done a great job of carefully reviewing and selecting outstanding articles. The new editorial board and I will continue the high standards they have set.

What has changed?

The editorial policy has changed. We have developed procedures and instructions to authors that will expedite the review and publication process.

Our new editorial policy (see below) requests that authors identify the value of their analysis or research effort described in their paper. Authors must submit a statement of contribution and, for application articles, a letter from a decision-maker stating the benefits of the analysis or research.

The articles submitted to the journal cover many military operations research problem domains and methodologies. In order to assign the most appropriate reviewer, we have identified application areas and methodologies. We have also expanded the number of Associate Editors to insure we have expertise in all of these areas. In addition, we have developed procedures to insure timely review of submitted papers. To help expedite the publication process, we have developed instructions for *Military Operations Research* authors (see below).

EDITORIAL POLICY

The title of our journal is *Military Operations Research.* We are interested in publishing articles that describe *operations research* (OR) methodologies used in important *military* applications. We specifically invite papers that are significant military applications of OR methodologies. Of particular interest are papers that present case studies showing innovative OR applications, apply OR to major policy issues, introduce interesting new problem areas, highlight educational issues, and document the history of military OR. Papers should be readable with a level of mathematics appropriate for a master's program in OR.

All submissions must include a statement of the major contribution. For applications articles, authors are requested to submit a **letter** to the Editor—exerpts to be published with the paper—from a **senior decision-maker** (government or industry) stating the benefits received from the analysis described in the paper.

To facilitate the review process, authors are requested to categorize their articles by application area and OR methodology, as described by the following lists. Additional categories may be added. (We use the MORS working groups as our applications areas and our list of methodologies are those typically taught in most graduate programs.)

INSTRUCTIONS TO *MILITARY OPERATIONS RESEARCH* AUTHORS

The purpose of the "instructions to *Military Operations Research* authors" is to expedite the review and publication process. If you have any questions, please contact Mr. **Michael Cronin**, MORS Editorial Assistant (email: morsoffice@aol.com).

Composite Group	APPLICATION AREA
I. STRATEGIC	Strategic Operations
	Arms Control
	Revolution in Military Affairs
II. NAVAL WARFARE	Expeditionary Warfare/Power Projection Ashore
	Littoral Warfare/Regional Sea Control
III. AIRLAND CONTINGENCY OPERATIONS	Missile Defense
	NBC Defense
	Mobility
	Air Warfare
	Land Warfare
	Spec Ops/Ops other than War
	Air Defense
	EW & Countermeasures
	Joint Campaign Analysis
IV. SPACE/C3I	C3
	Mil Environmental Factors
	Oper Cont of Space Systems
	OR and Intelligence
V. RESEARCH & DEVELOPMENT	Measures of Effectiveness
	Test & Evaluation
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VI. RESOURCES & READINESS	Soft Factors
	Social Science Methods
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	Readiness

OR METHODOLOGY
Deterministic Operations Research
Dynamic Programming
Inventory
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Multiobjective Optimization
Network Methods
Nonlinear Programming
Probabilistic Operations Research
Decision Analysis
Markov Processes
Reliability
Simulation
Stochastic Processes
Queuing Theory
Applied Statistics
Categorical Data Analysis
Forecasting/Time Series
Multivariate Analysis
Neural Networks
Nonparametric Statistics
Pattern Recognition
Response Surface Methodology
Others
Advanced Computing
Advanced Distributed Systems (DIS)
Cost Analysis
Wargaming

General

Authors should submit their manuscripts (3 copies) to:

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Military Operations Research Society
101 South Whiting Street, Suite 202
Alexandria, VA 22304

The manuscript should have camera ready illustrations and an electronic version of the manuscript prepared in WordPerfect or Microsoft Word. Per the editorial policy, please provide:

- authors statement of contribution (briefly describe the major contribution of the article)
- letter from senior decision-maker (application articles only)
- military OR application area(s)
- OR methodology (ies)

Length of Papers

Submissions will normally range from 5-25 pages (double spaced, 12 pitch, including illustrations). Exceptions will be made for applications articles submitted with a senior decision-maker letter signed by the Secretary of Defense.

Figures, Graphs and Charts

Please include camera-ready copies of all figures, graphs and charts. The figure should be of sufficient size for the reproduced letters and numbers to be legible. Each illustration must have a caption and a number which orders the placement of the illustration.

Mathematical and Symbolic Expressions

Authors should put mathematical and symbolic expressions in WordPerfect or Microsoft Word equations. Lengthy expressions should be avoided.

Approval of Release

All submissions must be unclassified and be accompanied by release statements where appropriate. By submitting a paper for review, an author certifies that the manuscript has been cleared for publication, is not copyrighted, has not been accepted for publication in any other publication, and is not under review elsewhere. All authors will be required to sign a copyright agreement with MORS.

Abbreviations and Acronyms

Abbreviations and acronyms (A&A) must be identified at their first appearance in the text. The abbreviation or acronym should follow in parentheses the first appearance of the full name. To help the general reader, authors should minimize their use of acronyms. A list of acronyms should be provided with the manuscript.

Footnotes

We do not use footnotes. Parenthetical material may be incorporated into a notes section at the end of the text, before the acknowledgment and references sections. Notes are designated by a superscript letter at the end of the sentence.

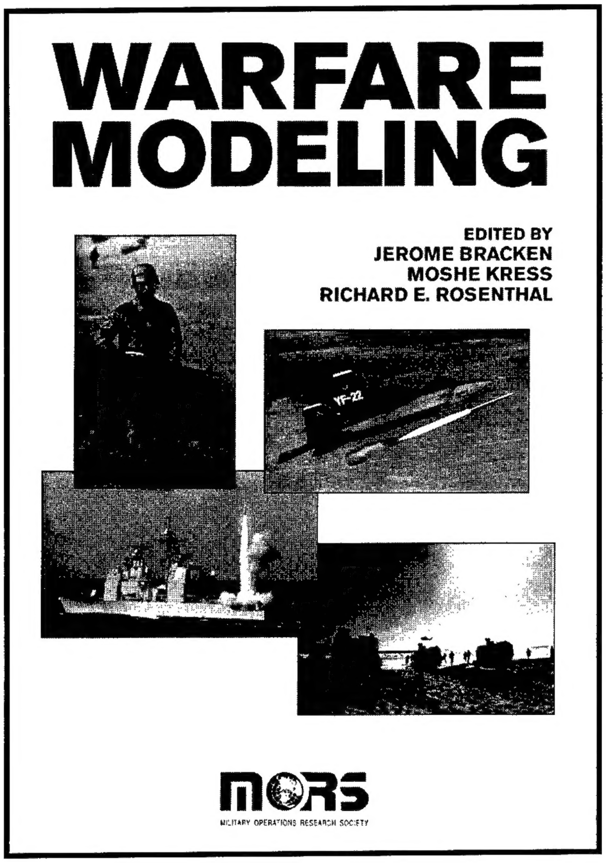
References

References should appear at the end of the paper and be unnumbered and listed in alphabetical order by the name of the first author.

POTENTIAL PAPERS OR SUGGESTIONS FOR THE JOURNAL

Military Operations Research is your journal. I need your help to identify the best articles for submission to the journal! If you have questions about a potential paper or suggestions for articles, please send me e-mail at gsparnell@aol.com.

I'm looking forward to seeing your article in *Military Operations Research*!



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The Military Operations Research Society (MORS) serves the students, developers, practitioners and clients of military operations research through the ongoing production of high-quality, low-cost meetings, books and periodicals.

Warfare Modeling was selected for publication by MORS because it contains 25 state-of-the-art chapters, addressing a wide class of operations research models that are of central importance in military planning, analysis and operations. Research on this subject matter seldom appears in the open literature. Most of it appears in reports and documents of government agencies and advisory corporations. This book is an invaluable reference for military OR professionals as well as for a more general audience of researchers and practitioners. The scope of models included—analytic and simulative, stochastic and deterministic, simple and complex, applied and theoretical, domestic and international—is very broad.

Warfare Modeling was edited by Jerome Bracken of Yale University, Moshe Kress of Israel's Center for Military Analyses, and Richard E. Rosenthal of the US Naval Postgraduate School. The Foreword is by Wayne P. Hughes, Jr., a retired Navy captain and MORS Fellow.

The book is comprised of seven sections:

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Executive Summaries

MILITARY OPERATIONS RESEARCH: RESPONDING TO CHANGE

by LtCol Paul F. Auclair
Edward F. Mykytka
and Dr. Gregory S. Parnell

For the 61st Military Operations Research Symposium, the Program Chair, Dan Barker, developed a survey to assess how the military operations research (MOR) community was responding to change. The survey was mailed in late 1992 (three years after the fall of the Berlin Wall) and over 1200 of the 3500 surveys were returned. After the symposium, Paul Auclair, Ed Mykytka, and Greg Parnell analyzed the survey results for the journal. The article provides a descriptive summary of the membership, assesses the need for professional education, and identifies key insights about how the focus of the community had changed by late 1992.

BENCHMARKING AND EFFICIENT PROGRAM DELIVERY FOR THE DEPARTMENT OF DEFENSE'S BUSINESS-LIKE ACTIVITIES

by Dr. William F. Bowlin

In recent years, the concept of total quality management (TQM) has been infused into many government operations. A key component of TQM is benchmarking. William F. (Bud) Bowlin illustrates an internal benchmarking approach using a methodology called data envelopment analysis (DEA) and Department of Defense commissary data. By using DEA, an analyst is able to identify superior performance, quantify performance gaps, and contribute to continuous improvement.

BOMB DAMAGE ASSESSMENT AND SORTIE REQUIREMENTS

by Dennis K. Evans

Computer programs that simulate warfare generally do not model bomb damage assessment (BDA) in a complex fashion. Most models either assume perfect BDA or no BDA, or allow the user to select between these two options. Assuming perfect BDA will allow a given force to destroy a given target set much more rapidly than will be the case if there is no BDA. It would superficially appear that perfect BDA and no BDA are opposite limiting extremes. This is not the case. The opposite limiting extremes are actually perfect BDA and extremely bad BDA. "No BDA" is an intermediate case. Hence, it is probably more reasonable to assume no BDA than perfect BDA in doing computer modeling. It is easy to simulate perfect BDA or nonexistent BDA, but any method of modeling imperfect BDA will be highly dependent on the scenario assumed. This suggests that "no BDA" may be preferable to "flawed BDA" as an assumption in computer modeling.

MODELING COST AND SCHEDULE UNCERTAINTIES—A WORK BREAKDOWN STRUCTURE PERSPECTIVE

by Paul R. Garvey

When cost and schedule uncertainty analyses are presented to decision-makers, questions asked with increased urgency are: What is the likelihood of achieving cost and schedule? What is the chance of exceeding the most likely cost for a given schedule? How are cost reserve recommendations driven by schedule uncertainties? Questions such as these are not readily answered by current analysis tools and techniques. This paper advances the state-of-the-practice by describing a family of, less known, multivariate probability models suitable for addressing these and related questions. The application of these models is presented from a work breakdown structure perspective, the traditional framework for developing program resource estimates.

DYNAMICAL INSTABILITY IN COMBAT MODELS: COMPUTER ARITHMETIC AND MATHEMATICAL MODELS OF ATTRITION AND REINFORCEMENT

by Dr. Julian I. Palmore

Foremost, this paper reports a methodology that can be used to assess information gain and the value of information in combat operations. Secondly, the experiments discussed in this report give modest insight into underlying relationships between tactical intelligence information and combat results. Finally, the paper reports a unique application of information theory to measure intelligence information concerning the size and disposition of enemy forces.

Combat models, nonlinear deterministic models of decision making processes, that deal with attrition of opposing forces, may contain dynamical instabilities and structural variance. Chaos in computation is one cause of instabilities in computer simulations of combat. Another cause of instabilities in decision processes is attributed to timing problems that arise when thresholds are crossed. An example of computer arithmetic effects for Patriot missile software is given in which very small timing errors accumulate and cause gross errors in detection and ranging. Several simple mathematical models of attrition and reinforcement are given and analyzed for chaotic behavior and nonlinear effects. Nonmonotonicity is demonstrated in the response of battle outcomes to changes in resources.

MILITARY TRAINING RESOURCE SCHEDULING: SYSTEM MODEL, OPTIMAL AND HEURISTIC DECISION PROCESSES

by LTC Mike McGinnis,
Professor Emmanuel Fernandez-Gaucherand
and Dr. Pitu B. Mirchandani

The United States Army trains thousands of new soldiers each year to fill vacancies in Army organizations. Initial entry training consists of two sequential phases: Basic Combat Training followed by Advanced Individual Training. Until recently, manual heuristic methods were used to schedule hundreds of training companies for initial entry training. We present two approaches for scheduling training resources for the Basic Combat Training phase of initial training. One is a dynamic programming decision model for optimally scheduling training resources. The second is a heuristic procedure implemented in an decision support system (DSS). Computational results are given for both approaches. The heuristic provides "good" solutions in terms of three performance measures: training quality as measured by an instructor-to-trainee ratio, resource utilization, and training costs.

Military Operations Research: Responding to Change

LtCol Paul F. Auclair
Edward F. Mykytka
Department of Operational Sciences
Air Force Institute of Technology
Pauclair@afit.af.mil
emykytka@afit.af.mil

Dr. Gregory S. Parnell
Department of Mathematical Sciences
Virginia Commonwealth University
gsparnell@aol.com

Application Areas:
Soft Factors

OR Methodology:
Statistics

ABSTRACT

This paper provides a summary of some top level insights obtained from analysis of a survey of military operations research practitioners conducted by the Military Operations Research Society in late 1992 through early 1993. In this paper, data obtained from the survey is used to provide a descriptive summary of the military operations research community and how it was responding to changes in the military and political environments at the time the survey was conducted.

INTRODUCTION

In preparation for the 61st Military Operations Research Society (MORS) Symposium, military operations research practitioners were surveyed in late 1992 through early 1993 to assess how the military operations research (MOR) community was responding to change. Of approximately 3,500 surveys distributed, 1,240 were completed and returned. The responses contain a great deal of information about the analysts who replied, the types of analyses they performed, and the organizations they worked for.

The survey data could easily support analysis of many detailed questions and comparisons. An initial summary was presented at the symposium held at the Air Force Institute of Technology in June 1993 by Mr. Dan Barker, the symposium chair. The purpose of this paper is to highlight some of the top-level insights gleaned from analysis of the survey data.

THE SURVEY

The survey was written and distributed by Barker in support of the symposium theme, "Responding to Change." It contains 17 questions concerning the demographics of the respondents, the nature of their work, and their perceptions of organizational changes and world events.

(The survey is included in the Appendix. To obtain a electronic copy of the response data [which can be read either as a dBase or Microsoft Excel file], send your request to MORS at the address indicated on the inside front cover of this journal.)

THE SURVEY POPULATION

The survey was distributed to those individuals who were members of MORS during 1992. One becomes a member of MORS by attending any one of its scheduled meetings. Attendance at a meeting assures membership in the society for three years. Thus, those individuals who attended at least one MORS-sponsored meeting since 1989 were mailed a survey. Although the respondents represent a sizable proportion (35%) of the MORS membership, they do not necessarily reflect the larger population of military operations research practitioners who, for one reason or another, did not participate in MORS-sponsored activities in the preceding three years.

Figure 1 compares the proportions of the MORS population affiliated with various agencies to the proportions exhibited by the survey respondents. As seen in this figure, the proportions are comparable. In fact, the proportions differ by less than five percentage points for any affiliation category.

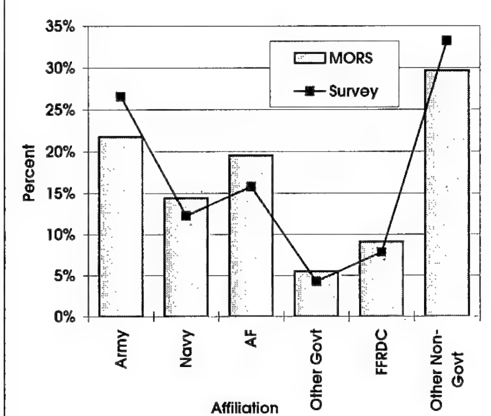

Figure 1: Survey Respondents vs. MORS Membership

Figure 2 illustrates that the survey respondents were a mix of private sector, civil service, and military analysts. Private sector civilians accounted for 38 percent of the responses, civil service employees made up 39 percent, and military analysts rounded out the final 23 percent. Unfortunately, we were unable to compare the proportions presented in Figure 2 with those of the MORS membership as a whole since the latter were not available from the society.

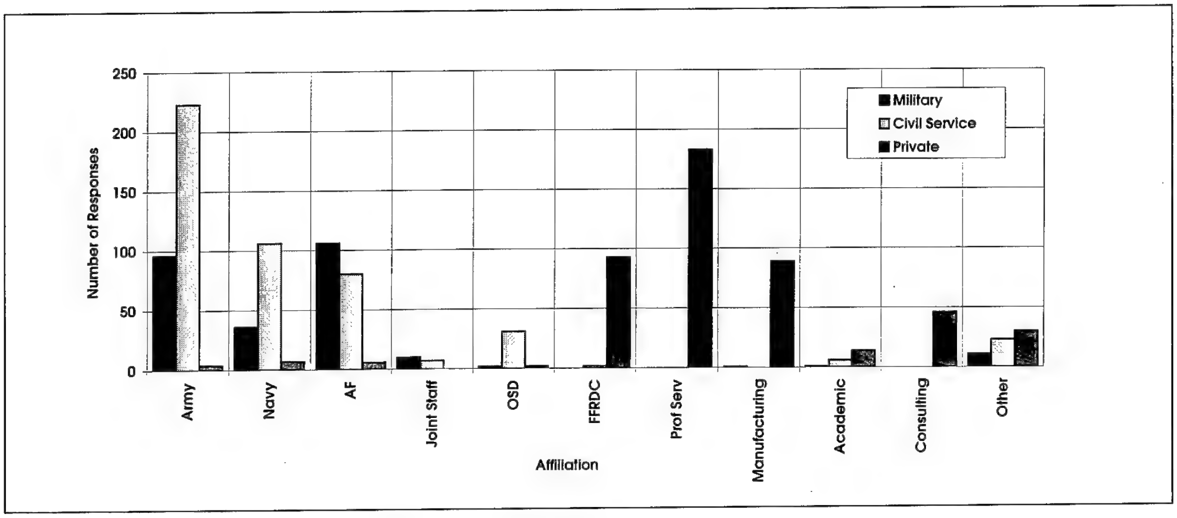

Figure 2: Affiliations of Respondents

Together, Figures 1 and 2 suggest that the survey population fairly represented the various agencies that comprised the MORS membership in early 1993 and that no single constituency dominated the responses. Therefore, we would expect the survey responses to be representative of the opinions, attitudes, and characteristics of the MORS population.

THE ANALYSTS

Experience

Table 1 and Figure 3 depict the distribution of analytical experience for the respondents in five-year increments.

Table 1: Years of Experience in Analysis

Years of Experience	Number of Respondents			
	Military	Civil Service	Private	Total
>25	0	93	94	187
21 – 25	5	85	87	177
16 – 20	19	80	74	173
11 – 15	44	67	103	214
6 – 10	67	91	71	229
≤ 5	110	47	41	198
Overall	245	463	470	1178

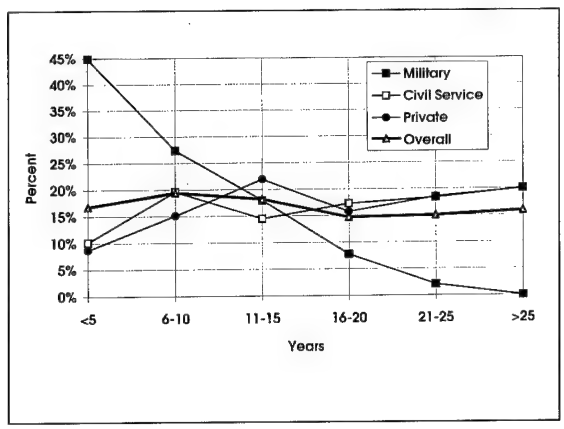

Figure 3: Analysis Experience

Although experience levels are distributed fairly evenly in the aggregate (with roughly the same number in each five-year group), there are substantial differences by affiliation. Likely because military careers are relatively short, nearly half of the military analysts had less than 5 years of analytical experience, 90 percent had less than 15 years experience, while those with over 20 years of analytical experience were rare. In contrast, a relatively small proportion of civilian analysts had less than 5 years of experience, and a relatively large number had well over 20 years of such experience.

Regardless of affiliation, however, the vast majority of respondents had spent most of their analytical careers focused on military operations research, as shown in Table 2.

Table 2: Military vs. Analytical Experience

Years of Military OR Exp.	Years of Analytical Experience					
	1–5	6–10	11–15	16–20	21–25	> 25
> 25						102
21–25					96	28
16–20				109	34	28
11–15			130	36	26	18
6–10		190	55	21	18	11
1–5	208	47	30	14	8	3
Overall	208	237	215	180	182	190

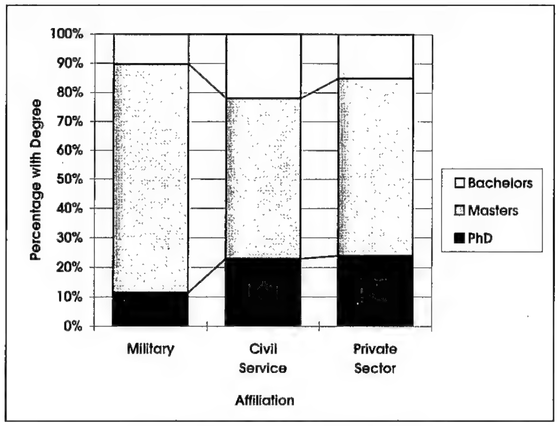

Figure 4: Education Levels

Education

The respondents reported a high level of education, with over 80 percent claiming at least a master's degree. Figure 4 depicts the proportions of population subgroups with various levels of education. Note the contrast in the distributions of degrees between the military and civilian groups. The military group has a slightly higher proportion of advanced degrees, but the proportion of civilians with doctoral degrees is more than double that of the military. We suspect that the value that the military places on graduate education tends to explain the higher proportion of military members with master's degrees while the shorter careers and applied focus of military analysts might account for the smaller proportion of military members with doctorates.

Self-Classification

Figure 5 depicts how the survey respondents indicated that they would describe their jobs to those unfamiliar with analysis. It is interesting to note the diversity of alternatives used by respondents to describe their discipline. Although the category "Systems Analyst" scored the highest, it did not constitute a majority of the responses. In fact, the second largest response was "Other," perhaps suggesting that other categories, such as "Cost Analyst" or "Economist," should have been included.

The results imply that military OR community has retained its original interdisciplinary nature-to the point that it is not dominated by any particular academic or professional affiliation. They could also imply that, being rather

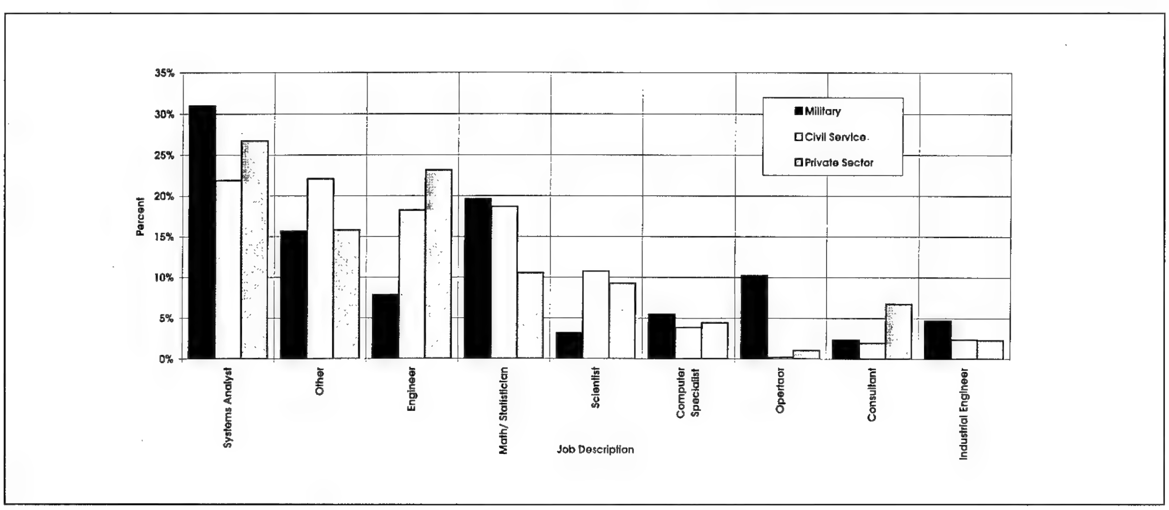

Figure 5: Alternate Job Descriptions

complex, interdisciplinary, and not popularly understood, military operations research is difficult to describe tersely to those not familiar with the discipline.

Training Needs & Activities

Despite the very high educational level observed, about half of the respondents felt they needed further education or training. The perceived need for additional training in analytical and computer techniques appeared to be a function of organizational affiliation. Figure 6 displays the proportion of respondents that indicated a need for such training within each organizational affiliation, ordered by the overall proportion expressing a need for further training. Overall, about 58 percent of the military analysts reported a perceived need for additional training in analytical and computer techniques; only about 38 percent of non-government civilians perceived need for such training. Government civilians fell between the two groups.

As seen in Figure 7, educational level related to the perceived need for additional training in analytical techniques. Not surprisingly, the higher the degree level, the lower the perceived need for further education or training in analytical techniques. On the other hand, educational level had less impact on the perceived need for additional training in computer techniques. As seen in Figure 7, the proportion of individuals expressing a need for additional training in computer techniques was approximately the same for those with either bachelors or masters degrees.

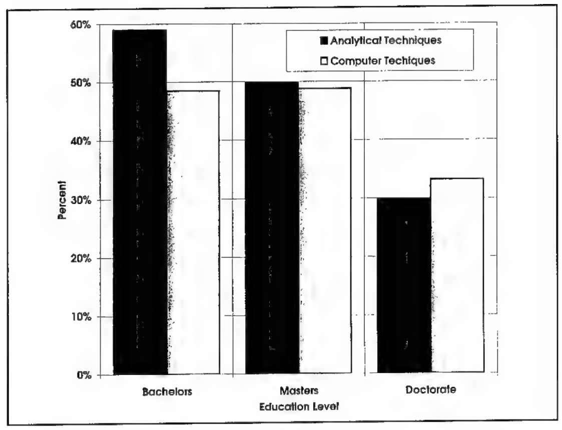

Figure 7: Perceived Training Needs by Education Level

Of those who felt they needed additional training, a sizable proportion (60%) was actively engaged in self study. Nearly a fifth (18%) of the respondents were currently enrolled in formal university courses. Overall, it seems that the respondents took education seriously, placed a high value on it, and expended the resources necessary to obtain it. This would seem to suggest that the MOR community was, on an individual level at least, attempting to respond to technical changes in the field.

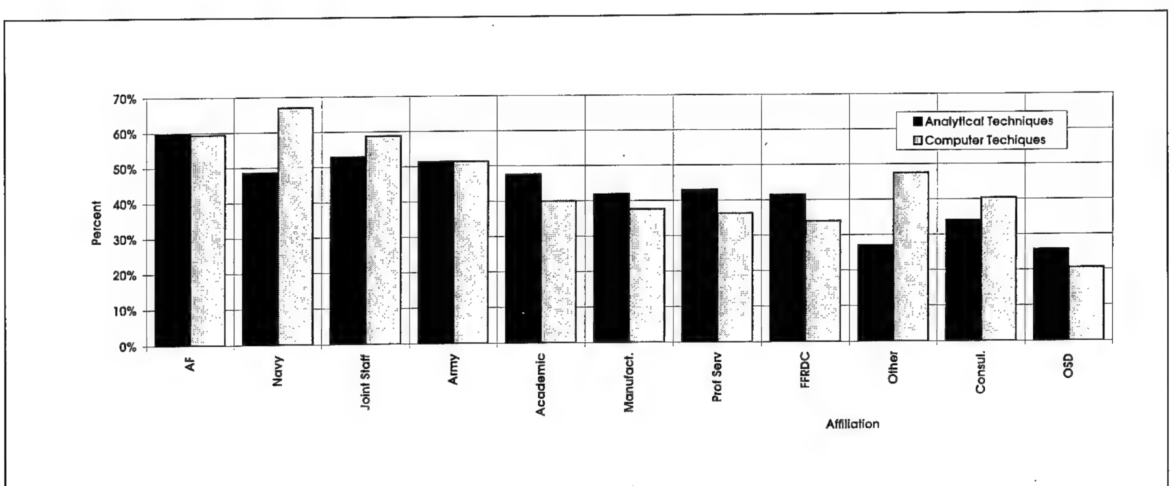

Figure 6: Perceived Training Needs by Affiliation

ANALYSIS

Functional Area Emphasis

To provide a broad perspective on where the MOR community perceived that it was, and expected it would be, investing much of its analytical effort, Figure 8 depicts the areas of primary analytical emphasis of the respondents (without regard to organizational affiliation) during the periods 1982-87, 1988-92, and projected for 1993-97.

The analysis of system or individual performance tends to dominate all other areas. Note, however, that campaign analyses are broken into conventional and strategic components and, taken together, represent a healthy interest in force effectiveness studies.

Of additional interest are those categories that are not primary areas of emphasis. For all periods considered, Operations & Maintenance (O&M), readiness, and sustainability have not been the primary area of emphasis of many analysts. We can infer that those areas have clearly not constituted the most important issues addressed by the MOR community. We should be careful not to conclude that those areas have not received sufficient attention. The survey does not address the appropriateness of the level of effort invested in each area; it only identifies the proportion of analysts who consider particular areas as dominant in their own work. Nevertheless, those less emphasized areas might have a great deal to do with our warfighting capability and interoperability. The level of effort afforded them might warrant reconsideration.

Theater Emphasis

While the primary areas of emphasis indicated a greater focus on campaign and force structure cost analysis, there was less effort devoted to specific theaters of operation. Figure 9 depicts the primary theaters of analysis among the respondents during the periods 1982-87, 1988-92, and projected for 1993-97.

This figure makes it very clear that, regardless of time period considered, most analysts do not focus on a primary theater. For analysts working primarily on a particular theater, there has been a noticeable decrease in those focusing on the Central European theater. Not surprisingly, there has been much more work in the Southwest Asia theater in recent years. On the other hand, despite events in Somalia and Eastern Europe, those theaters had attracted relatively little attention at the time the survey was administered.

Potential Near-Term Threats

Figure 10 displays the survey respondents' perceptions of the severity of various threats to national security for the five year period beginning in 1993. The threats were classified as "extremely serious" or as "one of the two most serious."

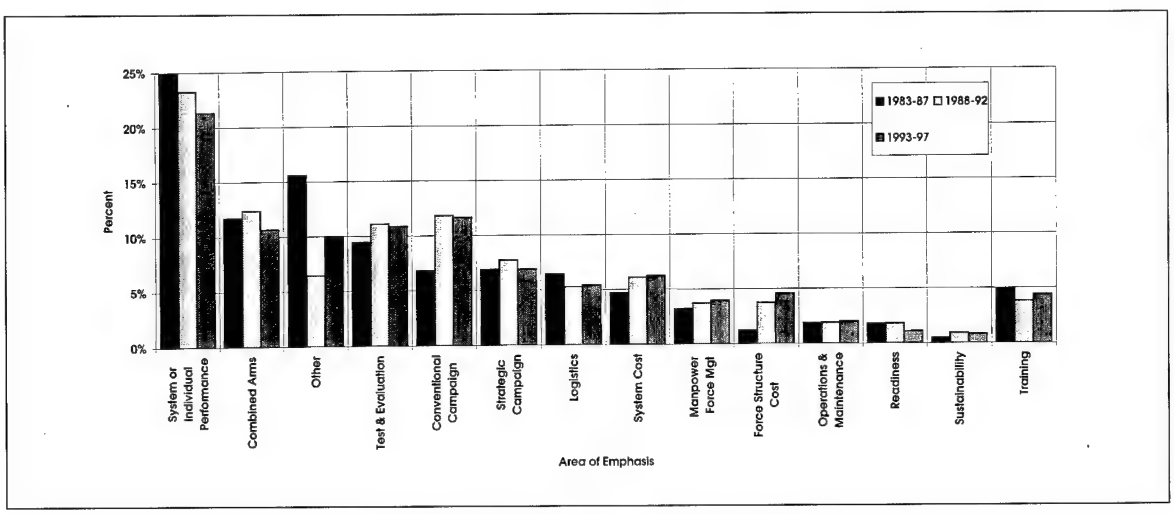

Figure 8: Primary Areas of Analysis

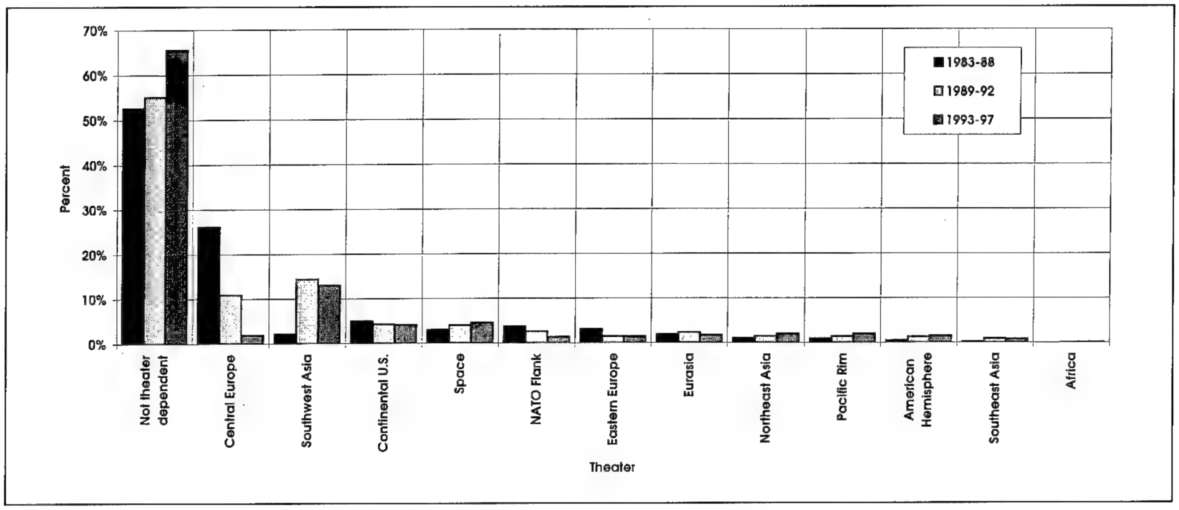

Figure 9: Primary Theater of Analysis

The most serious perceived threat to US national security was third world use of nuclear weapons, followed by our own federal budget deficit. It is interesting to observe the mix of purely military and social threats to national security. If MOR analysis were to respond to all perceived threats to national security, members of the MOR community could very likely be expected to study and analyze issues related to the budget deficit, health care alternatives, the environment, and novel applications of military forces in addition to traditional military topics. (Survey respondents also gave several "write-in" responses, with most such threats relating to the use of chemical or biological weapons.)

ORGANIZATIONS

The survey asked the individual respondents a number of questions about their analytical organizations. By virtue of the fact that respondents were individuals, rather than organizations, the survey is not likely to reflect the proportions of organizations exhibiting the characteristics expressed by the respondents. Rather, the results represent the proportions of the individual respondents who have certain perceptions about their organizations.

Most of the respondents (63%) believed that recent events had prompted their organizations to change. A similar proportion (65%) felt that

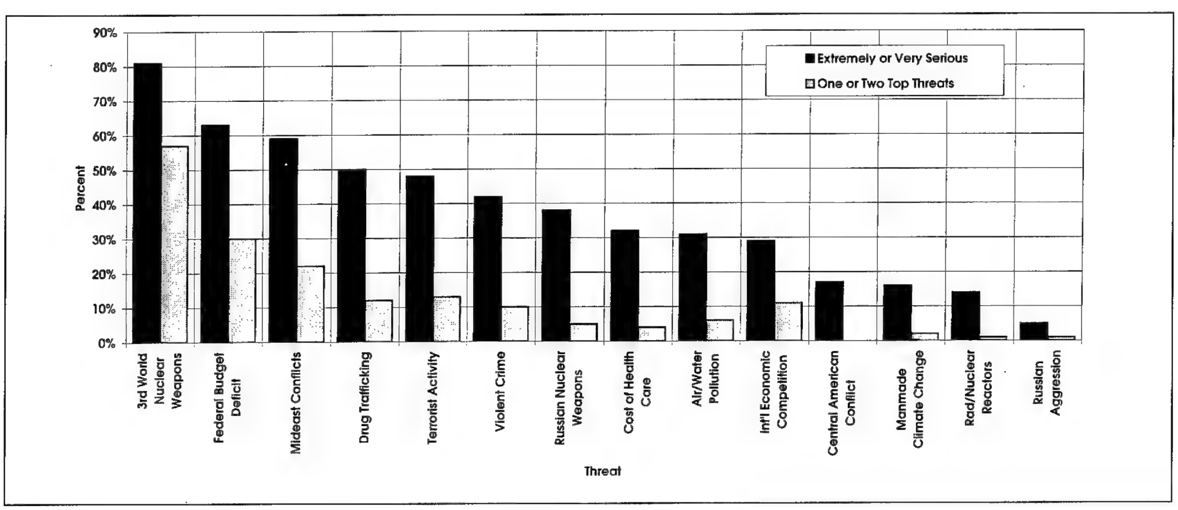

Figure 10: Potentials Threats

decision-makers were asking more of them in 1993 than during the 1983-1987 period. Despite a large number of individuals perceiving change in their organizations, that change was not uniformly evidenced in terms of staff reductions.

Figure 11 shows the relative proportions of individuals belonging to organizations that had changed the sizes of their staffs. In government circles, a greater proportion of Army respondents had seen their organizations downsized (military 68%, civil service 77%). The Air Force (military 60%, civil service 57%) and the Navy (military 36%, civil service 46%) follow. Significantly fewer individuals in private organizations (39%) had experienced such down-sizing.

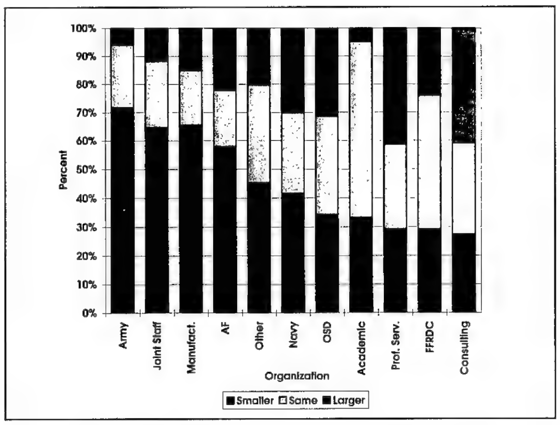

Figure 11: Right-Sizing Profile by Organization

SUMMARY

The survey's intent was to provide insight on the MOR community's response to change. When asked directly, 66% of the survey respondents thought the military analytic community was indeed responding to changes. Furthermore, we believe that the survey responses indicate that the MOR community has the intrinsic capability to be responsive.

The survey respondents characterized the MOR community as well experienced, highly educated, and interdisciplinary. We perceive military OR community as containing a healthy distribution of experience, with a good mix of newcomers, who tend to question dogma and time-honored approaches, and senior members, who provide the insights learned from their experience and corporate memory.

The community is also remarkably well educated and, apparently, eager to continue to learn. Even with 84 percent of the respondents having earned advanced degrees, 18 percent were enrolled in university courses, and 60% were involved in self-study. The community has also retained its original interdisciplinary nature. It is well-rounded in terms of professional and organizational affiliations, providing multiple perspectives on issues of national importance.

Clearly, the military OR community has the ingredients: experience, education, and an interdisciplinary focus: to effectively respond to change. The real issue is whether or not the community is living up to that potential. The survey provided only a few clues to the community's actual adaptiveness. The shift in primary theaters of interest indicates a responsiveness to changing world conditions, and the change in primary areas of interest reflects a growing emphasis on the effectiveness of integrated forces at the joint and theater levels. Even the ranking of the most significant threats to national security reflects the respondents recognition of a drastically different socio-political environment. These indicators imply that the MOR community is, indeed, responding to change. The survey, however, did not address the ultimate effectiveness of MOR in responding to change.

Military Operations Research is most useful when it is an integral component of the decision process. Is the MOR community illuminating issues, defining alternatives, and focusing attention on the right measures so that the military commanders and national policy decision makers can make the best decisions possible with the available data? That question can be answered effectively only by the consumers of MOR analysis. If the community continues to build on its heritage of satisfying the problem-solving needs of its customers, MOR will do more than simply respond to change. It will be in strong demand and have a major role in defining the changes of the future. Simply put, our future is up to us.

APPENDIX

The following is a summary of the questions posed to the MORS membership in the survey.

Who Are We?

1a. How many years have you been an analyst?

1b. Of these, have many have focused on MOR?

1c. How much longer do you anticipate being active in military analysis?

2. What is your affiliation with the military?

- US Army (military or civil service?)
- US Navy (military or civil service?)
- US Air Force (military or civil service?)
- Joint Staff (military or civil service?)
- OSD (military or civil service?)
- FFRDC
- Professional Services Firm
- Manufacturing Firm
- Academic
- Consultant
- Other

3a. Education-Highest Level Attained:

- High School
- Bachelors
- Bachelors Plus
- Masters
- Masters Plus
- Doctorate

3b. Gender (male or female)

3c. How do you describe your job to people unfamiliar with analysis?

- Engineer
- Scientist
- Mathematician/Statistician
- Computer Specialist
- Consultant
- Industrial Engineer
- Operator (pilot, infantry, etc.)
- Systems Analyst
- Other

What is Our Focus?

4. Indicate your primary area of analysis in each of the time periods 1983-87, 1988-92, and 1993-97.

- System or Individual Performance
- Combined Arms or Unit Effectiveness
- Conventional Campaign Contribution
- Strategic Campaign Contribution
- System Cost
- Force Structure Cost
- Test & Evaluation
- Manpower or Force Management
- Operations & Maintenance
- Logistics
- Readiness
- Sustainability
- Training
- Other

5. Indicate your primary theater of analysis in each of the time periods 1983-87, 1988-92, and 1993-97.

- Central Europe
- NATO Flank
- Eastern Europe
- Southwest Asia
- Eurasia
- Northeast Asia
- Southeast Asia
- Pacific Rim
- Africa
- American Hemisphere
- Continental United States
- Space
- Not Theater Dependent

6. Indicate the primary time frame considered in your own analyses in each of the time periods 1983-87, 1988-92, and 1993-97.

- Historical Perspective
- Immediate Application
- Less than 1 year
- 1 to 5 years

- Greater than 5 years
- No Particular Time Frame

7. Indicate the primary factor that limits the utility of your analysis by the decision maker in each of the time periods 1983-87, 1988-92, and 1993-97.

   - Accuracy
   - Precision
   - Presentation
   - Robustness
   - Timeliness

8. Indicate your own primary need today within each factor that limits the utility of your analysis.

   Factors:
   - Accuracy
   - Precision
   - Presentation
   - Robustness
   - Timeliness

   Needs:
   - Scenarios
   - Accepted Assumptions
   - Analytic Techniques
   - Data
   - Software
   - Hardware

9. What is your prognosis (excellent, good, poor, bad) for you seeing significant improvements in each of the following?

   - Scenarios
   - Accepted Assumptions
   - Analytic Techniques
   - Data
   - Software
   - Hardware

10. Do you need formal training to learn:

    - Analytical Techniques?
    - Computer Techniques?
    - Functional Knowledge?

11. If you need more training:

    - is it available from your organization?
    - are you taking university courses?
    - are you working on self study?

How Has the Threat Changed?

12. Thinking about the next 5 years or so, which of the following are extremely serious threats to our national security interests? Which are the one or two greatest threats?

    - International Drug Trafficking
    - Possession of Nuclear Weapons by Third World Countries & Terrorists
    - Environmental Problems
    - Violent Crime
    - High Cost of Health Care
    - Federal Budget Deficit
    - Economic Competition from Japan and Europe
    - Terrorist Activities around the World
    - Manmade Changes in Global Climate
    - Radiation from Old Nuclear Reactors & Nuclear Waste
    - Armed Conflicts in the Middle East
    - Armed Conflicts in Central America
    - Russian Nuclear Weapons
    - Russian Aggression Around the World

Have We Changed?

13. In your opinion, is the military analytic community responding to the changes from the last few years?

14. Are decision makers asking more or less of you than during the 1993-87 time period?

15. Have the changes in recent years caused your local analysis community to reorganize?

16. If so, are you more or less centralized as a separate department with an analysis identity?

17. Is your organization getting larger or smaller?

RIST PRIZE CALL FOR PAPERS

MORS offers two prizes for best papers—the Barchi Prize and the Rist Prize. The Rist Prize will be awarded to the best paper in military operations research submitted in response to this Call for Papers. The Barchi Prize will be awarded to the best paper from the entire 65th symposium, including Working Groups, Composite Groups, and General Sessions.

David Rist Prize: Papers submitted in response to this call will be eligible for consideration for the **Rist Prize**. The committee will select the prize-winning paper from those submitted and award the prize at the 66th MORSS. If selected, the author(s) will be invited to present the paper at the 66th MORSS and to prepare it for publication in the MORS journal, *Military Operations Research*. The cash prize is $1000. To be considered, the paper must be mailed to the MORS office and postmarked no later than **September 30th, 1997.** Please send the original, three copies and the disk.

Richard H. Barchi Prize: Author(s) of those papers selected as the best from their respective Working Group or Composite Group, and those of the General Sessions at the 65th MORSS will be invited to submit the paper for consideration for the **Barchi Prize**. The committee will select the prize-winning paper from among those presented, nominated and submitted. The prize will be presented at the 66th MORSS. The cash prize is $1000. To be considered, the paper must be mailed to the MORS office and postmarked no later than November 28, 1997. Please send the original, three copies and a disk.

Prize Criteria

The criteria for selection for both prizes are valuable guidelines for presentation and/or submission of any MORS paper. To be eligible for either award, a paper must, at a minimum:

- Be original and a self-contained contribution to systems analysis or operations research;
- Demonstrate an application of analysis or methodology, either actual or prospective;
- Prove recognizable new insight into the problem or its solution; and
- Not previously been awarded either the Rist Prize or the Barchi Prize (the same paper may compete for but cannot win both prizes.)

Eligible papers are judged according to the following criteria:

Professional Quality

- Problem definition
- Citation of related work
- Description of approach
- Statement of assumptions
- Explanation of methodology
- Analysis of data and sources
- Sensitivity of analyses (where appropriate)
- Logical development of analysis and conclusions
- Summary of presentation and results

Contribution to Military Operations Research

- Importance of problem
- Contribution to insight or solution of the problem
- Power of generality of the result
- Originality and innovation

Benchmarking and Efficient Program Delivery for the Department of Defense's Business-Like Activities[1]

Dr. William F. Bowlin
Department of Accounting
University of Northern Iowa
bud.bowlin@uni.edu

ABSTRACT

This paper reports on a prototype performance evaluation or benchmarking approach for Department of Defense (DoD) business-like operations using data envelopment analysis (DEA). These activities do not have a pure profit or "bottom-line" indicator of performance, and therefore, performance must be evaluated by some other means. The study illustrates a methodology for identifying best practices, providing benchmarks, quantifying performance gaps, and improving management control and efficiency for these types of operations via using data for the DoD commissary system. Although this study focuses on DoD commissaries, the technique is equally applicable to other government activities.

I. INTRODUCTION

The Air Force, Army, Navy, Marine Corps, and other Department of Defense (DoD) agencies operate several business-like activities for the benefit, morale, welfare, and recreation of military members and their dependents. Examples of these activities are commissaries (grocery stores), exchanges (department stores), noncommissioned officers clubs (restaurants), child development centers, golf courses, and bowling centers. These activities generate revenues of over $15 billion and are referred to as business-like since they provide services similar to many commercial businesses. In addition, they are similar to businesses in that they need to continuously improve and operate efficiently if they are to be successful in their endeavors. However, they differ from commercial enterprises in that they are primarily intended to provide a service to the military member, do not have a pure profit motive, and their operating revenues (e.g., patron fees, dues, and sales) are subsidized by Congress through appropriations.

In recent years, total quality management (TQM) has been infused into the operations of many businesses and government operations. A key component of TQM is benchmarking. Benchmarking is the identification of best practices leading to continuous improvement and more efficient operations.

The purpose of this study is to demonstrate the applicability of DEA as a benchmarking tool to aid DoD business-like activities in identifying superior performance, quantifying performance gaps, and other benchmarking considerations. Commissaries are selected as the organization to be evaluated because of the intense interest by Congress and others in reducing the appropriated fund support for commissaries and in making them more efficient. The analysis identifies, to management and others, areas that may be most suitable for commissary improvement. Thus, this information becomes the catalyst for change and continuous improvement required by TQM concepts.

In recent years, there have been several studies on the Military Services' commissary operations. Generally, the genesis of these studies has been concern over the amount of appropriated funds used to support the commissary systems and the desire to find a more efficient and effective way of securing the commissary benefit for the military member. The President's Commission on Privatization [1988] recommended that the private-sector manage and operate the military commissaries. The Jones Commission study [1989] resulted in the consolidation of the individual Service's commissary systems into the Defense Commissary Agency (DECA) in 1991. Finally, a project, Unit Cost Resourcing, has been initiated to improve productivity by measuring productivity with a single input to single output ratio—total appropriated dollar cost per dollar sales [Shycoff, 1990].

Following this introduction, the paper is organized in the following manner. Next is a section briefly discussing benchmarking and describing the methodology used in this research to measure commissary performance, i.e., data envelopment analysis (DEA). This is followed by a section which describes the data used in this study. Finally, there is a discussion of the results of the analysis, including insights into management control of commissary operations and possible benchmarks available from the analysis.

II. METHODOLOGY

Benchmarking

Benchmarking refers to the practice of searching for and identifying the best practices of a particular class or type of

Application Areas:
Resources, Performance Measurement

OR Methodology:
Deterministic Operations Research, Data Envelopment Analysis

organization. These "best operations" then become the benchmark by which the performance of similar activities is evaluated. The focus is on comparing an organization's current performance with the very best that has actually been achieved.

Once the activities with the best performance are identified, they can be analyzed to assist in identifying techniques, processes, and procedures that can be used to increase the performance, e.g., effectiveness or efficiency, of a similar activity. The source of these referenced best-practices organizations may be internal, i.e., from another part of the company, or external, i.e., from a competitor. Each source has its own advantages and limitations. Internal comparisons are used in this study for the reasons outlined below.

Spendolini (1992, p. 16) and Camp (1989, p. 61) recognize that internal benchmarking is a good, first-step in benchmarking investigations. Before starting an external benchmarking study, it is important that an organization thoroughly understand its own operations and identifies the critical issues that will be faced during an external investigation. This is accomplished with an internal benchmarking approach.

An internal comparison offers three additional advantages: (1) the relevant data needed for benchmarking are normally readily available or can be made available, with the right management emphasis, to initiate such a study, (2) data elements are consistently defined, and (3) management more readily accepts the results because they accept the comparability of the benchmark partners (the comparison organizations).

On the other hand, limitations of an internal comparison include that it runs the risk of identifying the best practices of a poorly run organization and not taking advantage of new ideas generated by others. To compensate for this problem, a comparison to competitors or to other external organizations may be desirable. However, the relevant data can be difficult, if not impossible, to obtain, data elements may not be consistently defined, and acceptance by management may be constrained because of the belief that "they aren't like us." An attempt was made to obtain comparable data from commercial grocery chains so that external benchmarking could be done for this study but the companies were not interested in participating.

Using data envelopment analysis, we identify "best-in-class" performance through internal benchmarking. In addition to identifying the best performing commissaries, DEA also provides some insights into why those commissaries are the best performers. As described in the next section, it identifies performance gaps in the input variables for each poorer performing commissary. DEA also identifies the superior performing commissaries to which the managers of poorer performing commissaries should turn to for assistance in identifying their operational shortcomings, e.g., identifying enablers.

Data Envelopment Analysis

Data envelopment analysis was developed by Charnes, Cooper, and Rhodes (CCR) [1978] for measuring efficiency of not-for-profit entities. Since its inception, it has been further developed, extended, and applied in a variety of ways and settings [Banker *et. al.*, 1989]. A good summary of the technical computational details of the methodology and additional description of the various DEA models can be found in several references including Callen [1991] and Banker *et. al.* [1989].

There are several variations of the DEA model. Here we take a closer look at the CCR fractional programming version of the DEA model because it more clearly provides insights into the characterizations of the DEA model and the reasons for its appropriateness as a benchmarking tool. This model is as follows:

$$\text{(1) maximize } ho = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$\text{subject to: } \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, j=1,\ldots,n$$

$$\frac{u_r}{\sum_{i=1}^{m} v_i x_{io}} \geq \epsilon, r=1,\ldots,s$$

$$\frac{v_i}{\sum_{i=1}^{m} v_i x_{io}} \geq \epsilon, i=1,\ldots,m$$

where:

h_o = The performance rating sought for organization "o" which is the one member of the reference set, $j=1,\ldots,n$, that is to be evaluated relative to the others.

v_i = The variable weights for each type of input, $i=1,\ldots,m$, which will be determined by the solution of the model.

u_r = The variable weights for each type of output, $r=1,\ldots,s$, which will be determined by solving the model.

x_{io} = The known amount of input "i" used by organization "o" during the period of observation.

y_{ro} = The known amount of output "r" produced by organization "o" during the period of observation.

x_{ij} = The known amount of input "i" used by organization "j" during the period of observation.

y_{rj} = The known amount of output "r" produced by organization "j" during the period of observation.

ϵ = A positive constant introduced to insure that all of the observed inputs and outputs will have some positive value assigned to them.

In the CCR ratio form, the problem is interpreted as one of choosing u_r and v_i values, subject to the constraints, that will assign the maximum possible ratio value, h_o, which is to be used as the performance rating of organization "o". The essential aspect of the CCR ratio construction is the reduction of the multiple output, multiple input situation for each organization to that of a single "virtual" output and a single "virtual" input for each organization. Formula (1)'s objective then is to maximize the ratio of virtual output to virtual input for each organization in the population under evaluation.

The multipliers or weights, u and v, are determined by the model directly from the data and not assigned a priori. A "best" set of weights is determined for each of the organizations to be evaluated. Given this best set of weights, each organization "o" is evaluated on whether any other organization "j" achieved a higher ratio than organization "o" using the latter's best weights. DEA thus identifies the subset of organizations that have the best virtual output to virtual input ratio and thereby, determines a performance frontier based on those "best" performing organizations.

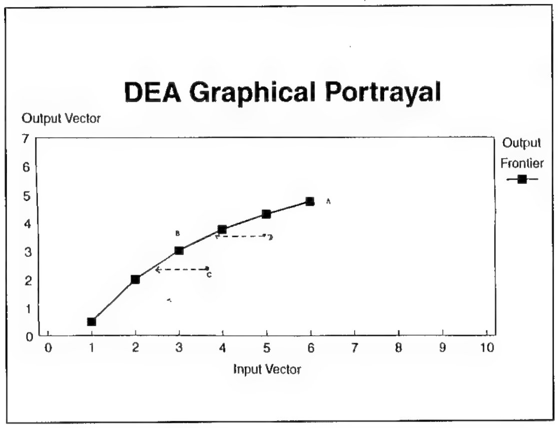

Figure 1: DEA Graphical Representation

Figure 1 is a simplified graphical representation of DEA. Y is a vector of multiple outputs which become the virtual output and X is a vector of multiple inputs which become the virtual input. DEA creates a frontier (curve A) based on observed data for the "best" organizations, i.e., the organizations with the best ratio of multiple outputs (the virtual output) to multiple inputs (the virtual input). These frontier organizations (e.g., point B) are best performers, receive a rating of one, and are used to assess the performance of other similar, but less productive operations. The less productive organizations (e.g., points C and D) receive a rating of less than one depending on their distance from the frontier.

DEA has several characteristics, described in the following paragraphs, which make it suited as a benchmarking tool. First, DEA is empirically based. That is, the solution to the model is based on observed data and behavior. DEA does not require a priori identification of the underlying functional form to relate the outputs and inputs nor the "weights" (u and v) to indicate the

relative importance of each output and input used to determine the solution to the model. Thus, the solution to the model is not based on some hypothetical or theoretical construct but on actual performance and must conform to the actual performance of an entity. This makes the results of the model more acceptable to managers which is an important consideration for a benchmarking model. In addition, not requiring a priori identification of a production function is particularly advantageous for this study since an appropriate production function has not yet been developed for commissaries.

To get a better understanding of this characteristic, it is sometimes helpful to compare DEA to index number and regression approaches. Because of the absence of any need for prescribing the underlying functional form, weights etc., in an a priori manner, DEA contrasts with indexes or regression approaches which are dependent on numerous assumptions and, in principle, require a great deal of analytical theorizing prior to choosing the form(s) in which they are used. However, in reality, the actual, observed forms are variable in nature—possibly differing for each organization under evaluation. Consequently, with DEA more flexibility is permitted in the form of the actual production function employed by each organization under evaluation than is true with regression and index number approaches.

Second, DEA is an extremal methodology which optimizes on each observation to estimate a performance frontier by coming as close as possible to each observation. That is, DEA employs "n" optimizations, one for each entity being evaluated. Consequently, DEA not only distinguishes between best performing and inferior or average performing operations, it also provides information on what weaker operations can do to move to higher levels of performance. The best performing organizations become the benchmark (comparison set of organizations) for judging the performance of the other organizations and provide the basis for DEA to determine the levels of output and input the unit under evaluation should achieve to be on the performance frontier. These efficient levels of output and input are the goals for this organization developed from best-practices organizations. In benchmarking terminology, the difference between the DEA computed performance and the actual performance observed is the performance gap.

Note also that DEA's frontier orientation differs from the interior or average orientation embodied in least-squares regression. Least-squares regression analysis uses a single optimization to come as close as possible to all of these points. Thus, least-squares regression tends towards determining an average performance and does not make any separations between good and bad performance.

Third, DEA handles multiple output and multiple inputs simultaneously and in a manner that recognizes the simultaneous variations and interactions between the many components. Thus, the resulting rating is a type of total-factor productivity measure. A total-factor productivity measure is desired since single or partial measures, such as output per unit of labor input (labor productivity) or output per unit of capital input (capital productivity), seldom capture the multiple interests, goals, or values of the entity being evaluated. Therefore, these alternative measures may provide incomplete and misleading indications of performance. This is an apparent limitation of the single measure used in the Unit Cost Resourcing project referenced above.

Fourth, as shown in Ali and Seiford [1990], because of DEA's optimization process, the units of measure for the input and output variables do not influence the performance measure computed as long as the unit of measure for each variable is the same for all organizations. Therefore, the variables can be measured in dollar or nondollar units. Additionally, even if all variables are measured in the same units (e.g., dollars), the variables still maintain their individual identities.

Finally, the aggregate scalar efficiency rating computed by the CCR DEA model can be further separated into scalar measures of technical and scale efficiencies [Banker, 1984 and Banker et. al., 1984]. Technical efficiency is attained only if the outputs cannot be increased or inputs decreased without decreasing some output or increasing some input. In other words, technical efficiency exists when output is the greatest amount possible for the input amounts consumed. Scale efficiency exists when an organization is producing at the optimum scale, i.e., a

change in the inputs results in a proportionate change in the output (constant returns to scale).

The above discussion describes the attributes that make DEA attractive and suitable for this particular research. However, there are model limitations that should be kept in mind. First, DEA requires that all inputs and outputs have positive values. Thus, if a variable has a zero or negative value, it must be adjusted. There are techniques for doing this and, generally, this will not cause a problem in the DEA evaluation because the model is unaffected by the units of measure as long as they are consistent for each variable across all organizations included in the evaluation set.

Second, the data must consist of a relatively homogeneous set of organizations, and there needs to be three observations (organizations) for each variable used in the model in order to have sufficient degrees of freedom for a valid evaluation. With less than three observations per variable, the model will tend to classify or rate each organization as performing 100% efficient because it will heavily weight the one variable that is most advantageous to each organization.

Finally, because of DEA's optimizing principle, it cannot be used, as regression can, for general characterizations of the data nor for prediction of future behavior of the entire collection of observations. Additionally, since there is no random error term, the model does not directly consider that the performance (i.e., output attained or input consumed) may be an anomaly and not representative of normal operations. Furthermore, no statistical test of significance can be applied to the "weights."

IV. DATA

Outputs and Inputs

In this study, we use data that were being reported by each of the military services to the Department of Defense—one output, sales dollars, and three inputs, personnel work-years, store square footage, and appropriated fund expenditures. For a preliminary study such as this one, it is desirous and advantageous to use data that are understood and accepted by management so that management can more readily relate to and understand the results. Thus, since these data are being collected, reported, and used, management understands them and has accepted them as being informative and relevant in evaluating the performance of the organization. The input and output data are limited but sufficient to capture a significant portion of efforts and accomplishments of commissaries.

Sales. Sales is one measure of a commissary's accomplishments. It is what the commissaries are attempting to produce and provides an overall indication of how well the commissary is doing to include partially measuring how well individual commissaries are satisfying its customers. It is an aggregate measure that represents the customer's sense of the balance between cost and value received. The greater the level of sales (for a given level of input), the better and more effective the commissary is in satisfying its customers. Since sales level (in dollars) is the only output measure currently being reported by the military commissary system, it is the only output used in this study.

Work-years (WY). This input measures the amount of labor used in producing the sales. The greater the number of work-years, the greater the amount of sales we would expect. It includes both part-time and full-time civilian and military employees. The part-time work-years are converted to full-time equivalents.

Square Feet (SQFT). This input variable is a surrogate for the capital (equipment and facilities) used in producing the sales. The greater the square footage, the larger the investment in the facility and equipment used in producing the sales. It is recognized that there are limitations in using this measure as a surrogate for the capital investment, e.g., it does not reflect the age of the equipment or facilities. However, it does capture the size aspect of the capital investment. In addition, although this variable is not directly controllable by local management, it does provide useful information relative to how well a fixed asset is being used and the appropriate scale of operations that is required for the particular market in which that commissary is operating.

Appropriated Funds (APF). APF are the funds provided by Congress to partially support the operation of commissaries. The amount of APF used in support of commissaries is a key concern of Congress and the military services. Efficient use of these funds is necessary to insure

continued Congressional support for and maintenance of this military-service benefit. The expected relationship between sales and APF would be the greater the amount of APF support, the greater the level of sales.

Substituting the variables into the DEA model from (1), we have:

$$(2)\ \text{maximize } h_o = \frac{u_s S_o}{v_{wy} WY_o + v_{sqft} SQFT_o + v_{apf} APF_o}$$

$$\text{subject to: } \frac{u_s S_1}{v_{wy} WY_1 + v_{sqft} SQFT_1 + v_{apf} APF_1} + \frac{u_s S_2}{v_{wy} WY_2 + v_{sqft} SQFT_2 + v_{apf} APF_2} + \ldots \frac{u_s S_{237}}{v_{wy} WY_{237} + v_{sqft} SQFT_{237} + v_{apf} APF_{237}} \leq 1$$

$$\frac{u_s}{v_{wy} WY_o + v_{sqft} SQFT_o + v_{apf} APF_o} \geq \epsilon$$

$$\frac{v_{wy}}{v_{wy} WY_o + v_{sqft} SQFT_o + v_{apf} APF_o} \geq \epsilon$$

$$\frac{v_{sqft}}{v_{wy} WY_o + v_{sqft} SQFT_o + v_{apf} APF_o} \geq \epsilon$$

$$\frac{v_{apf}}{v_{wy} WY_o + v_{sqft} SQFT_o + v_{apf} APF_o} \geq \epsilon$$

where the symbols are as previously defined.

As can be noted, these variables capture and aggregate two commonly used measures of productivity: labor productivity, sales per workyear, and capital productivity, sales per square foot. Hence, although the data are limited, they are sufficient to provide us with ratios of the type commonly used to evaluate performance for commercial retail operations such as sales per square foot and sales per employee (per workyear in this case) along with a measure, sales per APF dollar, that is unique to the commissary operating environment.

Decision Making Units

Charnes, Cooper, and Rhodes [1978] define a decision making unit (DMU) as an organization or entity that has the authority and responsibility for managerial decision making. For this study, DMUs are the stores operated by the Army, Air Force, Navy, and Marine Corps in the continental United States plus Hawaii and Alaska. Commissaries operated in Europe, Japan, and other overseas areas are excluded due to having significantly different operating environments. There are 89 Air Force commissaries, 72 Army commissaries, 62 Navy commissaries, and 14 Marine Corps commissaries, for a total of 237 DMUs, analyzed in this study over a two-year period.

Data Collection

Fiscal years 1988 and 1989 data are used in this study, thereby allowing both time-series and cross-sectional analyses to be completed. The data are obtained from the DoD Commissary Operations Report (DD-FM&P(A) 1187) for each year. This report was prepared by each military service's commissary service and submitted to the Assistant Secretary of Defense (Force Management and Personnel).

Sales figures are standardized by inflating FY 1988 sales to FY 1989 levels using the producers price index. FY 1988 APF amounts are normalized to FY 1989 levels by using standard Department of Defense inflation rates.

V. ANALYSIS

This research consists of three primary investigations. The first analysis is of individual store performance and illustrates the type of benchmark information available from DEA for individual stores. Second, system-wide performance is reviewed and the impact of store size is examined so as to provide insight on potential improvement system-wide. The final part is a sensitivity analysis assessing the impact of

different variables on the reported results. Note that this is not a complete benchmarking study because we do not fully identify performance "enablers." The intent is to stimulate management into identifying opportunities for improving commissary operations.

Individual Store Performance

As previously discussed, DEA provides a summary measure of performance along with indicating the amount each input should be reduced (i.e., the performance gap) in order for the commissary to be operating on the performance frontier and therefore, operating at the same level as the best-performing commissaries. It also identifies the organizations on which the evaluated unit's performance rating is based. Table 1 illustrates, for Maxwell Air Force Base commissary, the management (benchmarking) information available from the data envelopment analysis. Note that the following conclusions are tempered by the need to insure that the variables used completely capture the accomplishments and efforts of the commissary system.

TABLE 1
MAXWELL AFB EFFICIENCY RESULTS

	1988	1989
a. Actual Variable Values		
Sales ($000)	$ 25,408	$ 26,097
Work-years	103	90.6
Square Feet	30,100	30,100
Appropriated Funds ($000)	$ 2,092	$ 2,175
b. Efficiency Ratings		
Technical Efficiency	.612	.699
Scale Efficiency	.997	.996
Aggregate Efficiency	.610	.696
c. Input Variable Values If Efficient For Actual Sales		
Work-years	63	63.3
Square Feet	18,400	21,000
Appropriated Funds ($000)	$ 1,218	$ 1,519
d. Technical Efficiency—Weights of Benchmark Units		
Fort Leonard Wood (1988)	.1302	.2799
Fort Belvoir (1988)	.2579	.1417
Pope Army Facility (1988)	.5294	.3919
Carswell Air Force Base (1989)	.0825	.1866
Total	1.0000	1.0000

Reviewing section b of Table 1, Maxwell is relatively technically inefficient with ratings of only 61% in 1988 and nearly 70% in 1989. Thus, there is a significant performance gap between Maxwell and the best-performing, frontier commissaries to which Maxwell is compared. Given Maxwell's sales levels in 1988 and 1989, DEA indicates that for Maxwell to operate efficiently, it needs to reduce its input consumption by 30%-39%. Section c of Table 1 provides the input levels that should have been used to produce the sales level achieved. Note that these values are the achievable performance goals that are expected from a benchmarking process based on the superior performance of the "best" organizations under review. For example, in FY 1988, Maxwell should have used only 63 work-years, 18,400 square feet, and $1,218 thousand in appropriated funds—compared to 103 work-years, 30,100 square feet, and $2,092 thousand (normalized to FY 1989 dollars) APF actually consumed. The difference between these superior performance levels and the actual level achieved represent the performance gap for Maxwell.

Although some of these findings may be the result of factors not controllable by management, e.g., square footage, they still indicate that performance gaps exist and input reductions are possible. The exact amount of the reductions necessary to bring this commissary in line with the best performing commissaries to which it was compared needs to be determined by management and may vary from those provided by DEA because of unique circumstances applicable to Maxwell or some anomaly in the operations of the commissary for those years. However, DEA does provide an indication of the magnitude of the reduction, e.g., 30%-39%. In addition, even though square footage is not controllable by management, DEA does indicate that the physical plant, as represented by the square footage, is too large and not being effectively used to generate sales.

Continuing the review of section b of Table 1, Maxwell is nearly scale efficient at close to 100% for both FYs 1988 (.997 or 99.7%) and 1989 (.996 or 99.6%). Thus, at its scale of operations, it nearly gets a proportionate change in its output for a change in its input, and it is producing at its optimum scale given the particular technical

efficiency conditions. Hence, there is little performance gap corresponding to Maxwell's scale of operations and very little improvement is possible in its scale of operations.

Finally, the units listed in section d of Table 1 constitute the comparison set or benchmarking units upon which Maxwell's performance rating is based. Recall that these comparison units are on the performance frontier, are considered to be the best performing commissaries, and would receive a DEA rating of one (1) or 100%. In addition, these commissaries have an output and input mix similar to Maxwell's and should be the first to be reviewed for ideas for further improvement in Maxwell's performance. Thus, these are the best-practice activities with which Maxwell should be initially compared.

Note that the value listed in section d for each comparison unit is not its efficiency rating. These numbers represent the weight of each unit's contribution, as objectively determined by the DEA model, in determining Maxwell's performance rating. For example, Pope Army Facility (1988) had the greatest influence on Maxwell's 1988 rating with a weight of .5294 which indicates that 52.94% of Maxwell's rating was based on the Pope Army Facility performance.

System-Wide Performance

Table 2 summarizes the DEA ratings by military service, type of efficiency (aggregate, technical, or scale), and fiscal year. For example, we see

TABLE 2

SUMMARY OF MEANS AND STANDARD DEVIATIONS FOR DEA PERFORMANCE RATINGS

	Air Force	Army	Navy	Marines
Aggregate Efficiency				
1988				
Mean	.61	.59	.52	.64
Standard Deviation	.13	.18	.17	.12
1989				
Mean	.68	.57	.53	.64
Standard Deviation	.15	.18	.17	.12
Technical efficiency				
1988				
Mean	.63	.64	.61	.68
Standard Deviation	.13	.16	.13	.10
1989				
Mean	.70	.61	.62	.67
Standard Deviation	.14	.16	.12	.10
Scale Efficiency				
1988				
Mean	.97	.92	.85	.94
Standard Deviation	.06	.10	.20	.05
1989				
Mean	.97	.93	.85	.94
Standard Deviation	.05	.11	.20	.06
Store Average Square Feet				
1988	22,981	17,816	14,107	14,069
1989	23,543	20,110	14,168	14,376

that the mean aggregate efficiency rating for Air Force commissaries in FY 1988 is .61 (61%). The Air Force's average technical efficiency rating is 63% and scale efficiency is 97% for FY 1988.

The average technical efficiency rating for all stores over the two-year period of this study is in the mid 60 percent range and the average scale efficiency rating is in the 90 percent range except for the Navy (see Table 2). Thus, based on the variables used in this study, improvements can be made in the technical and scale efficiencies of all the commissaries with much less improvement evident in the scale of operations.

However, we do not want to overemphasize what appears to be a low average technical efficiency rating. Recall that it is a relative rating based on the best performing commissaries. Senior commissary management may deem these superior performers as not being representative of normal or expected operations and therefore, may set lower performance goals than indicated by DEA.

Store Size

The relationship between store size and the efficiency ratings is examined to determine if certain sizes of stores are better performers or more efficient than others. Do larger commissaries perform better than smaller commissaries?

For this review, two input measures of size are used—store square footage and amount of appropriated fund (APF) usage. Sales are not used as a measure of size since they are an output variable in our model. The first test used is univariate correlation analysis. In this test, the DEA efficiency ratings are regressed against square footage and APF for each commissary for each year to determine whether there is a relationship between size of the operations and efficiency of operations, i.e., as stores become larger (smaller) do they become more or less efficient. Table 3, which provides the R^2 values for these tests, shows that the correlation analysis did not reveal a strong relationship between efficiency and either APF or store square footage for any of the Services. Thus, a linear relationship between the DEA efficiency ratings and the size of operations is not apparent.

TABLE 3

R^2 VALUES for CORRELATION ANALYSIS

	Air Force	Army	Navy	Marines
Technical Efficiency				
APF	.32	.17	.21	.25
Square Footage	.24	.11	.07	.36
Scale Efficiency				
APF	.22	.34	.36	.54
Square Footage	.31	.43	.45	.67

Since the univariate correlation analysis fails to show any relationship, we check for square footage and APF thresholds at which efficiency is significantly different. A statistical approach provided by Banker [1989 and 1993] is used to test whether there is a difference in the efficiency ratings between two groups of commissaries under consideration, i.e., larger commissaries versus smaller commissaries. The test statistic, as indicated below, is a measure of the variance around the reciprocal of the inefficiency ratings. Banker also indicates that the DEA inefficiency rating follows either a half-normal or exponential distribution. We assume that the performance ratings follow an exponential distribution because it has the appropriate skewness and nonnegativity characteristics associated with the DEA scalar measure.

The test statistic in this case is given by:

$$(3)\quad [\sum_{j \in N_1} (1/h^*_j - 1)/n_1]/[\sum_{j \in N_2} (1/h^*_j - 1)/n_2]$$

where:

h^*_j = The efficiency rating computed by the DEA model for each commissary in the reference group

n_1 = The number of commissaries included in group 1 (N_1)

n_2 = The number of commissaries included in group 2 (N_2)

This test statistic follows an F-distribution with $(2n_1, 2n_2)$ degrees of freedom. The justification for this test statistic can be found in Banker [1993].

No APF threshold could be identified at which efficiency ratings are significantly different. However, the findings for square footage are different. The greatest difference in performance ratings exist at the 20,000 square foot threshold. The stores with greater than 20,000 square feet are much more efficient at a level of significance of less than .00001 for both technical and scale efficiency. We further subdivided the upper level into a 20,000 to 30,000 square feet segment and another segment for greater than 30,000 square feet. We found that the stores with 20,000 to 30,000 square feet are more technically and scale efficient than those stores with under 20,000 square feet ($p = .0021$ and $p < .00001$ respectively). Those stores with greater than 30,000 square feet are more technically and scale efficient than stores with 20,000 to 30,000 square feet ($p = .0543$ and $p < .00001$ respectively). Thus, the evidence suggests that large stores are generally both more technically and scale efficient than small stores. However, we would not want to conclude that all stores should be greater than 30,000 square feet. There could be other factors not captured with our limited number of variables which would influence that decision. For example, stores of that size would probably be too large for the customer base at smaller installations. A measure of market size is discussed later in the paper.

These results also suggest that standardized policies and procedures for all stores across the Department of Defense may not be appropriate. Some smaller commissaries, e.g., Fort Story and Hill AFB, were considered efficient. The policies and procedures of these efficient smaller commissaries could be reviewed for possible adoption for smaller commissaries.

Sensitivity Analysis

Using Air Force data only (because of availability), the sensitivity of the individual commissary performance results to the addition of different inputs and outputs is examined. One variable tested, both as an output and an input, is average weekly operating hours. Operating hours, as an output variable, serves as a surrogate for customer service. We would expect that the greater the operating hours the greater the service to the customer and the greater the inputs, i.e., work-years and appropriated funds, consumed. Another analysis is accomplished using operating hours as an input—the more hours the store is open the greater the sales should be. Surprisingly, neither case made much difference in the efficiency ratings. Possibly, the information provided by operating hours is being captured in the other output and inputs.

A third sensitivity analysis is done using a measure of market size—the number of active duty military and retirees in the area—as an input. The population numbers were obtained from Economic Resource Impact Statements (ERIS) prepared by each Air Force base. This analysis did change the efficiency ratings of Air Force stores—with some stores getting better ratings while the ratings for other stores deteriorated. However, there was no pattern to these changes. This indicates that the DEA performance ratings are sensitive to and influenced by market size. Unfortunately, this information is not readily available for Army, Navy, and Marine Corps installations so no further analysis is pursued.

Future Research Relative to Commissaries

There are two areas of study that the Defense Commissary Agency (DECA) may wish to pursue based on this research. First, a longitudinal analysis would facilitate evaluating whether efficiencies and economies were achieved from the consolidation of the four military commissary systems into one DoD system. A second potential area of study is in developing additional output and input measures. In this research, output and input variables on which data were currently being gathered, reported, and accepted as useful by management were used. However, some key success factors may have been missed. For example, additional outputs better reflecting customer service could be developed and tracked. On the input side, a measure of market size might be useful, as indicated earlier in this study.

VI. CONCLUSIONS

This study evaluated a benchmarking approach for a DoD business-like, service activity. It served two purposes. First, the benchmarks identified provide possible avenues for improving commissary operations. Since commissary operations are a far more complex process than represented by this four variable model, the benchmarks indicate the direction that needs to be pursued rather than specific operational metrics that are immediately attainable. Also, the data used are from 1988-89 and may not be representative of today's operations. Department of Defense business-like activities such as commissaries need to be operated in an efficient manner so as to insure economical use of taxpayers' dollars, preserve the military members' benefits these dollars provide, and minimize military member and dependent costs (prices, fees, dues, etc.).

Second, the research illustrates and evaluates the potential for using data envelopment analysis (DEA) as a benchmarking or "best-practices" tool in order to improve performance and provide better management and cost control. DEA is an objective means for selecting benchmark partners, quantifying performance gaps, and identifying information which might be useful for improving performance.

DEA as a Benchmark Identifier

Benchmarking or best-practices analysis is an integral aspect of total quality management. It requires the identification and observation of best practices and then projecting future performance based on adopting these best practices. Thus, this process involves, among other actions, identifying benchmark partners (i.e., organizations with superior performance) as a source of comparative data or benchmarks, gathering performance data, determining any performance gap, projecting superior performance levels, and establishing realistic, achievable performance goals [Camp, 1989]. Are the benchmark partners better? By how much? What level of performance should we realistically expect?

In the case presented here, superior performance is identified via DEA. Then through using DEA, performance gaps are quantified and potential benchmarks for a commissary's continued improvement are provided. DEA indicates where improvements are necessary; e.g., less work-years, reduced appropriated fund expenditures, and less store square footage; and the magnitude of these improvements. In addition, DEA provides the benchmark units (stores operating at 100% efficiency) upon which a less efficient store's performance is determined. These comparison stores have a similar production function and can serve as benchmark partners for identifying best practices from which the inefficient store can get ideas for improvement.

In addition, by benchmarking the "best-of-class," the Defense Commissary Agency can focus on continuous improvement of processes and activities that support these processes. For example, this research indicates that larger stores (greater than 20,000 square feet) are significantly more efficient than smaller stores. Hence, DECA may wish to initially focus its efforts on the small commissaries in order to improve system efficiency. There are some small commissaries that are efficient. The operating procedures of these commissaries should be examined and possibly adopted for use by the other small less efficient stores—even if they aren't in the comparison set of the inefficient store.

Finally, a key aspect of the benchmarking process is gaining management acceptance of the findings. Contributing to this acceptance is benchmarks that are relevant and current. DEA provides objective, empirically-based performance evaluations based on attained performance. Thus, through DEA, benchmark information can be collected that will allow setting performance objectives that are realistic and attainable. Furthermore, studies have shown via theoretically constructed (artificially generated) data and actual operational data that DEA is able to identify superior performance and establish realistic performance goals [e.g., Banker et. al., 1988 and Bowlin et. al., 1985].

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END NOTES

[1] December 1994, Revised: March 1996, Revised: April 1996. The views expressed herein are those of the author and do not necessarily reflect the views of the Department of Defense.

Bomb Damage Assessment and Sortie Requirements

Dennis K. Evans
Office of the Secretary of Defense Program Analysis and Evaluation

ABSTRACT

Computer programs that simulate warfare generally do not model bomb damage assessment (BDA) in a complex fashion. Most computer models either assume perfect BDA or no BDA, or allow the user to select between these two options (sometimes globally and sometimes on a target-by-target basis). Assuming perfect BDA will allow a given force to destroy a given target set much more rapidly than will be the case if there is no BDA, because perfect BDA prevents the waste of weapons on targets that have already been destroyed. It would superficially appear that perfect BDA and no BDA are opposite limiting extremes. This is not the case. The opposite limiting extremes are actually perfect BDA and extremely bad BDA. "No BDA" is an intermediate case, although probably closer to "worst case" than to "best case" in most realistic battlefield scenarios. Hence, it is probably more reasonable to assume no BDA than perfect BDA in doing computer modeling. It is easy to simulate perfect BDA or nonexistent BDA, but it is very hard to model flawed BDA. Any method of modeling imperfect BDA will be highly dependent on the scenario assumed.

INTRODUCTION

Computer programs that simulate warfare generally do not model BDA in a complex fashion. Most computer models either assume perfect BDA or no BDA, or allow the user to select between these two options (sometimes globally and sometimes on a target-by-target basis). Assuming perfect BDA will allow a given force to destroy a given target set much more rapidly than will be the case if there is no BDA, because perfect BDA prevents the waste of weapons on targets that have already been destroyed. In this paper, we attempt to develop a mathematical formalism for determining the effects that BDA can have on the number of sorties launched against a given target set. We also address the following two questions:

- Are perfect BDA and no BDA opposite limiting extremes?
- Is it generally more reasonable to assume perfect BDA or no BDA in a computer simulation of war?

In order to facilitate this analysis, we examine two variations on one basic scenario, which is described below.

BASIC SCENARIO:

Suppose that we have N_0 live identical point targets initially present. We launch n raids against these targets, where n is constrained to be a positive integer. In each raid, there is one sortie against each target that is thought to be "live." All sorties are identical. On the first raid, all N_0 targets are accurately known to be alive, so there is one sortie against each of the N_0 targets, for a total of N_0 sorties. Let P denote the probability that any one sortie kills the target that it is attacking. We constrain P to be less than one; otherwise the problem is trivial with all N_0 targets being destroyed on the first raid. Beginning after the first raid, there is a reconnaissance flight to assess the damage done on the previous raid. The first reconnaissance flight will examine all N_0 targets, because all N_0 targets were attacked on the first raid.

Make the following definitions:

N_0 = number of targets initially present
N_j = expected number of targets still "living" after j raids
k_i = expected number of targets killed on raid number i
K_j = expected cumulative number of targets killed in first j raids = $N_0 - N_j$
P = single-sortie kill probability (one aircraft versus one target)
p = probability of wrongly assessing a dead target as being living
1-p = probability of correctly assessing a dead target as being dead
q = probability of wrongly assessing a live target as being dead
1-q = probability of correctly assessing a live target as being live
T_j = expected target set remaining after raid j = expected target set for raid j+1 = expected number of sorties on raid j+1
A_j = expected number of live targets assessed as alive after j raids
D_j = expected number of dead targets assessed as alive after j raids
S_j = expected total number of sorties flown in first j raids

Application Areas:
Conventional Force Analyses

OR Methodologies:
Linear Programming

It is immediately clear that:

$$K_j = \sum_{i=1}^{j} k_i \qquad (1)$$

$$S_j = \sum_{i=0}^{j-1} T_i \qquad (2)$$

$T_o = A_o = N_o$

$k_o = 0$

$K_1 = k_1 = N_oP$

$N_1 = N_o(1-P) = N_o - k_1$

T_1 is the target set for raid number two. T_1 consists of that portion of N_1 which is correctly assessed as being alive plus that portion of k_1 which is incorrectly assessed as being alive. In other words, $T_1 = A_1 + D_1$. We find that:

$$T_1 = N_0(1-P)(1-q) + N_0Pp \qquad (3)$$

Things get more complicated when we come to the second and later raids. We find that:

$$T_j = A_j + D_j \qquad (4)$$

If $p = 0$, then $D_j = 0$ and $T_j = A_j$.

$$k_j = PA_{j-1} \qquad (5)$$

The remainder of the analysis depends on what happens after each reconnaissance flight.

EXAMPLE ONE

Two basic limiting cases are possible. *In the limiting case considered here, any target that is (rightly or wrongly) assessed as "dead" after any raid will never be assessed or attacked again.* Any target that is assessed as "live" after all raids to date will be attacked again, by one sortie, on the next raid. Generally speaking, the number of sorties per raid will decline as the number of targets assessed to be "live" decreases.[1] In this limiting case, we find that:

$$A_j = N_0(1-q)^j(1-P)^j \qquad (6)$$

$$k_j = PA_{j-1} = N_0P(1-q)^{j-1}(1-P)^{j-1} \qquad (7)$$

$$K_j = \sum_{i=1}^{j} k_i = N_0P\sum_{i=1}^{j} (1-q)^{i-1}(1-P)^{i-1} = \frac{N_0P[1-(1-q)^j(1-P)^j]}{1-(1-q)(1-P)} \qquad (8)$$

Letting the number of raids approach infinity, we find that:

$$\lim_{j \to \infty} K_j = \frac{N_0P}{1-(1-q)(1-P)} \qquad (9)$$

The value of K_j can reach N_o only if $q = 0$. If $q = 1$, only the first raid destroys any targets.

We also have the following expression for D_j:

$$D_j = \sum_{i=1}^{j} k_i p^{j+1-i} = N_0P\sum_{i=0}^{j-1} (1-q)^i(1-P)^i p^{j-i} \qquad (10)$$

This simplifies to:

$$D_j = N_0Pp\left[\frac{p^j-(1-q)^j(1-P)^j}{p-(1-q)(1-P)}\right] \qquad (11)$$

Using Equations (3), (5), and (11), we find that:

$$T_j = N_0(1-P)^j(1-q)^j + N_0Pp\left[\frac{p^j-(1-q)^j(1-P)^j}{p-(1-q)(1-P)}\right] \qquad (12)$$

Rearranging Equation (12) and using Equation (2), we get:

$$S_j = \left[N_0 - \frac{N_0Pp}{p-(1-q)(1-P)}\right]\sum_{i=0}^{j-1} (1-q)^i(1-P)^i + \left[\frac{N_0Pp}{p-(1-q)(1-P)}\right]\sum_{i=0}^{j-1} p^i \qquad (13)$$

Evaluating the two summations, we get the following expression for S_j:

$$S_j = \left[N_0 - \frac{N_0Pp}{p-(1-q)(1-P)}\right]\frac{1-(1-q)^j(1-P)j}{1-(1-q)1-P)} + \left[\frac{N_0Pp}{p-(1-q)(1-P)}\right]\frac{1-p^j}{1-p} \qquad (14)$$

In order to determine the expected number of sorties required to achieve some damage goal, G, the first step is to set $K_n = G$ in Equation (8) and then solve for n. (If G is greater than the limiting value of K_n from Equation (9), then it is impossible to achieve the goal and this procedure will not work.) This determines the expected number, n, of raids required to reach the goal. The result is:

$$n = \frac{\ln\left[\dfrac{N_0P - G(1-(1-q)(1-P))}{N_0P}\right]}{\ln((1-q)(1-P))} \qquad (15)$$

The value of n predicted by Equation (15) will generally not be an integer; it should be rounded up. The next step is to use Equation (14) to determine the corresponding number of sorties.

Finally, we use Equations (8) and (14) to develop a relationship between K_j and S_j. The resulting expression is:

$$K_j = \frac{Pp - P(1-q)(1-P)}{p - Pp - (1-q)(1-P)}\left[S_j - \frac{N_0Pp}{p-(1-q)(1-P)}\left(\frac{1-p^j}{1-p}\right)\right] \qquad (16)$$

If $p = 0$, then Equation (16) reduces to $K_j = PS_j$. If $q = 0$ but $p > 0$, this does not lead to any major simplification.

NOTE: In the preceding analysis, T_j, K_j, and S_j can assume nonintegral values. In practice, when doing computer simulations of war, it might be reasonable to round T_j up or down to the nearest integer after every raid. Similarly, the expression for S_j should probably be rounded off to the nearest integer. Finally, it would probably be reasonable to classify the entire target set as dead once N_j drops below 1. Of course, it is usually the case that the goal is to kill something like 50% or 80% of the target set, so the question of how a computer program handles the last target, out of several thousand, is probably not important.

Best possible case: p = q = 0 (perfect BDA)

In this case, we find that:

$$T_j = N_o(1-P)^j$$

$$k_j = N_oP(1-P)^{j-1}$$

$$K_j = N_0P\sum_{i=1}^{j}(1-P)^{i-1} = N_0(1-(1-P)^j) = PS_j \qquad (17)$$

As the number of raids goes to infinity, K_j goes to N_o. Moreover, no weapons are ever wasted on dead targets and the ratio between the number of targets destroyed and the number of sorties flown is simply P.

Limiting case: q = 0 and p = 1 (no BDA)

If $q = 0$, there is no major simplification unless $p = 1$. If $p = 1$, there are N_o sorties in every raid and:

$$S_j = jN_0 \qquad (18)$$

$$K_j = N_0P\sum_{i=1}^{j}(1-P)^{i-1} = N_0(1-(1-P)^j) \qquad (19)$$

The expected fractional number of raids, n, needed to destroy a goal of G targets is given by:

$$n = \frac{\ln\left(1 - \dfrac{G}{N_0}\right)}{\ln(1-P)} \qquad (20)$$

The value of n from Equation (20) should be rounded up. The corresponding number of sorties is just nN_o. As the number of raids approaches infinity, K_j goes to N_o and all targets are destroyed. However, some sorties are wasted on dead targets.

Worst case: q = 1 and p = 1

In this case, N_oP targets are destroyed in the first raid. After the first raid, all dead targets are erroneously classified as being alive and all live targets are erroneously classified as being dead. All targets that survived the first raid are immune from further attacks, so the number of targets killed remains stuck at N_oP no matter how many raids are launched.

EXAMPLE TWO

We now consider a different limiting case. *Beginning after the first raid, there is a reconnaissance flight to assess the status of all N_o targets. Any target that is assessed as "dead" after any raid will not be attacked on the next raid.* Any target that is

assessed as "live" after any raid will be attacked again, by one sortie, on the next raid. *In this limit, a target that is assessed as being dead can get back into the target set at a later time.* This means that the target set will not automatically shrink from raid to raid. Equations (1) through (5) from the Basic Scenario are still valid. However, we have the following relationships, which were different from those used in Example One:

$$D_j = pK_j \tag{21}$$

$$A_j = (1-q)N_j \tag{22}$$

$$N_j = N_{j-1} - PA_{j-1} = N_{j-1} - k_j \tag{23}$$

Equations (22) and (23) immediately imply that:

$$N_j = (1 - P(1-q))N_{j-1}, \textit{for } j > 1 \tag{24}$$

We know that $N_1 = (1 - P)N_o$, so Equation (24) immediately leads to:

$$N_j = N_0(1 - P)[1 - P(1-q)]^{j-1}, \textit{for } j \geq 1 \tag{25}$$

$$A_j = N_0(1 - q)(1 - P)[1 - P(1-q)]^{j-1}, \textit{for } j \geq 1 \tag{26}$$

$$K_j = \sum_{i=1}^{j} k_i = k_1 + \sum_{i=2}^{j} k_i = PN_0 + P\sum_{i=2}^{j} A_{i-1} \tag{27}$$

The previous equation leads to:

$$K_j=PN_0+P\sum_{i=1}^{j-1} A_i=PN_0+N_0P(1-P)(1-q)\sum_{i=0}^{j-2}(1-P(1-q))^i \tag{28}$$

$$K_j=PN_0+PN_0(1-P)(1-q)\left[\frac{1-(1-P(1-q))^{j-1}}{P(1-q)}\right] \tag{29}$$

The previous equation simplifies to:

$$K_j = N_0 - N_0(1-P)(1-P(1-q))^{j-1} = N_0 - N_j \tag{30}$$

In this scenario, the number of targets killed approaches N_o as the number of raids approaches infinity, provided that $q < 1$, because a live target that is erroneously classed as dead will eventually get back into the target set. If we use the results of Equations (30) and (25), we find that $K_j + N_j = N_o$, which is a requirement that must be satisfied. Equations (4), (21), (26), and (30) imply that:

$$T_j = N_0p + N_0(1-P)[1-P(1-q)]^{j-1}(1-q-p), \textit{for } j \geq 1 \tag{31}$$

Using Equations (2) and (31), plus the expression for T_o, we find that:

$$S_j = N_0\left[1 + p(j-1) + (1-P)(1-q-p)\left(\frac{1-(1-P(1-q))^{j-1}}{P(1-q)}\right)\right] \tag{32}$$

In order to determine the expected fractional number of sorties required to achieve some damage goal, G, the first step is to set $K_n = G$ in Equation (30) and then solve for n. This determines the expected number, n, of raids required to reach the goal. The result is:

$$n = 1 + \frac{\ln\left[\dfrac{N_0 - G}{N_0(1-P)}\right]}{\ln(1-P(1-q))} \tag{33}$$

The value of n from Equation (33) should be rounded up. The next step is to use Equation (32) to determine the corresponding number of sorties.

Finally, we use Equations (30) and (32) to develop a relationship between K_j and S_j. The result is:

$$K_j = PN_0 + \left[\frac{P(1-q)}{1-q-p}\right](S_j + N_0(p-jp-1)) \tag{34}$$

If $p = 0$ and $q > 0$, then $K_j = PS_j$ for all q, but Equation (30) for K_j and Equation (33) for the number of raids required do not simplify any, and Equation (32) for S_j does not simplify much. If $q = 0$, then there is no major simplification except when $p = 1$.

NOTE: In the preceding analysis, T_j, K_j, and S_j can assume nonintegral values. Hence, the same caveats that were expressed about fractional sorties in Example One also apply here.

Best possible case: p = q = 0 (perfect BDA)

In this case, the limiting behavior is identical to that in Example One. Because no weapons are

ever wasted on dead targets whenever p = 0, the ratio between the number of sorties flown and the number of targets destroyed is simply P.

Limiting case: q = 0, p = 1 (no BDA)

In this case, Equations (18) through (20) from Example One are still valid. If q = 0 and 0 < p < 1, there is no major simplification and the limiting behavior is different from that in Example One.

Worst case: q = 1 and p = 1

In this case, N_0P targets are destroyed in the first raid and no targets are destroyed in the later raids, no matter how many raids are launched.

CONCLUSIONS

1) *The opposite limiting extremes are not "perfect BDA" and "no BDA." The opposite extremes are perfect BDA and extremely bad BDA. "No BDA" is an intermediate case, although probably closer to "worst case" than to "best case" in most realistic battlefield scenarios.*

2) *Most computer models assume either perfect BDA or no BDA, or allow the user to choose between these two options. Given that "no BDA" is an intermediate case, whereas "perfect BDA" is an extreme limit, it is generally more reasonable to choose "no BDA" if that option is available.*

3) *It is easy to simulate perfect BDA or nonexistent BDA, but it is hard to model flawed BDA. Any method of modeling imperfect BDA will be highly dependent on the scenario assumed.*

4) Of the computer models known to the author, only HEAVY ATTACK allows the user to select intermediate BDA. HEAVY ATTACK contains a BDA factor, y, that can be varied from zero to one, on a global basis. The HEAVY ATTACK damage accumulation formula is:

$$K_n = \left(\frac{N_0}{y}\right)\left(1-e^{-\frac{nPy}{N_0}}\right) \qquad (35)$$

In Equation (35), n is the number of sorties. In HEAVY ATTACK, each aircraft is limited to attacking one target, so the expected damage from each sortie is just P. If individual aircraft could attack more than one target, then P would be replaced by the total expected damage probability per sortie. If y = 0, then Equation (35) leads to linear damage addition (K_n = nP), which is what we would expect with perfect BDA. If y > 0 in HEAVY ATTACK, then damage accumulates more slowly than is the case if y = 0, but there is no obvious relationship between y and any of the parameters in this analysis. In fact, y may have no physical meaning; it may simply be an arbitrary factor that emulates the effects of poor BDA by making damage accumulate more slowly than would otherwise be the case. For example, a nonzero value of y reduces the amount of damage done on all raids, including the first raid. In reality, poor BDA should reduce the efficiency of later raids, but not the first raid.[2]

ENDNOTES

[1] An alternative approach would be to keep the number of sorties per raid constant. This means that each target that is assessed as being "live" after raid j would tend to be hit harder on raid j+1 than on the previous raid. We do not consider this alternative, because it would quickly lead to fractional sorties against each target assessed as live.

[2] *Survey and Description of USAF Conventional Munitions Allocation Models*, Major Kirk A. Yost, Office of Aerospace Studies, Directorate of Requirements, Air Force Materiel Command, 5 December 1994

Modeling Cost and Schedule Uncertainties—A Work Breakdown Structure Perspective

Paul R. Garvey

The MITRE Corporation, Bedford, Massachusetts
pgarvey@mitre.org

ABSTRACT

This paper presents an application of bivariate probability theory to modeling cost and schedule uncertainties. It has long been recognized that program cost and schedule are correlated; however, formal methods have not been developed in the cost analysis community to study their joint behavior. To address this, bivariate models for approximating the joint and conditional probabilities of program cost and schedule are presented. Specifically, the bivariate lognormal and bivariate normal-lognormal models are discussed. The statistical properties of these models are provided. A cost analysis application is presented to illustrate their use in a practical context.

I. INTRODUCTION

A common framework for quantifying the cost impacts of uncertainty is the work breakdown structure (WBS) [1]. The WBS is a product-oriented hierarchy of cost elements that define the program. Illustrated in table 1, many of these elements, such as system engineering, reflect distinct level-of-effort activities that vary over a program's schedule. Thus, the uncertainty associated with total program cost is correlated to the uncertainty in the overall schedule.

Numerous methods exist for quantifying cost uncertainty from a WBS perspective. However, they are focused on producing a univariate distribution of total cost; schedule uncertainties are not explicit. Cost-schedule questions that are probability-based (e.g., what is the chance of delivering the system within cost and schedule estimates?) are not readily answered by existing models, methodologies, or spreadsheet simulation applications.

This paper extends current practice by providing bivariate probability models from which joint and conditional distributions of cost and schedule can be developed. This fosters the early identification of cost-schedule risks such that mitigation strategies can be planned and implemented in a timely manner.

Table 1. An Illustrative WBS

Cost Element (*CE*)		Estimation Heuristic (Example Only)
Prime Mission Equipment (PME) Segment		
CE_1	Hardware (HW)	$HW_{Cost} = \sum_i HW_{i_{Cost}}$
CE_2	Software (SW)	$SW_{Cost} = 8.2\lambda_{SW}(Source\ Lines/1000)\epsilon$
CE_3	Integration & Assembly (I&A)	$I\&A_{Cost} = S_{I\&A}(\lambda_{I\&A})\left[(SL)_{Tool\ \&\ Test\ Equip} + (SL)_{HW \mid SW\ Integ}\right]$
System Support (SS) Segment		
CE_4	System Engineering (SE)	$SE_{Cost} = S_{Pgrm}\lambda_{SE}(SL)_{SE}$
CE_5	Program Management (PM)	$PM_{Cost} = S_{Pgrm}\lambda_{PM}(SL)_{PM}$
CE_6	Test & Evaluation (TE)	$TE_{Cost} = S_{Pgrm}\lambda_{TE}(SL)_{TE}$
$\vdots$		
CE_m	Training	$Training_{Cost} = \xi_{Training}HW_{Cost}$
CE_n	Data	$Data_{Cost} = \xi_{Data}(HW_{Cost} + SW_{Cost})$
$\vdots$		

Notes: $\lambda_{(\bullet)}$ denotes a labor rate; ϵ is a statistical error distribution; $S_{I\&A}$ denotes that portion of the overall program schedule S_{Prgm} for I&A activities; $SL_{(\bullet)}$ denotes a staff-level; and $\xi_{(\bullet)}$ denotes a cost factor.

Application Areas:
Resource Analysis,
Risk Analysis,
Decision-making Under Uncertainty

OR Methodology:
Probability theory

II. BACKGROUND & MODEL DOMAIN

This section provides a brief background on the types of uncertainties present in military systems engineering projects. A discussion on how these uncertainties relate to the models described in this paper is presented.

A. Background

Uncertainty analysis had its genesis in a field known as military systems analysis [2], which was founded in the 1950's at the RAND Corporation. Shortly after World War II, military systems analysis was developed to aid defense planners with long-range decisions on force structure, force composition, and future theaters of operation. Cost became a critical consideration in military systems analysis models and decision criteria. However, cost estimates of future military systems, particularly in the early acquisition phases, were often significantly lower than the actual cost or an estimate developed at a later phase. In *Cost Considerations in Systems Analysis* [3], Gene H. Fisher attributes this difference to the presence of uncertainty; specifically, cost estimation uncertainty and requirements uncertainty.

Cost estimation uncertainty originates from errors in cost-schedule estimation models, from the misuse or misinterpretation of cost-schedule data, or from misapplied cost-schedule estimation methodologies. Economic uncertainties that affect the cost of technology (e.g., inflation/deflation), the labor force, or geopolitical policies further contributes to cost estimation uncertainty.

Requirements uncertainty originates changes to a system's specified configuration (i.e., the system architecture). Different system configurations can arise from changes in the understood threat, mission objective, or strategic outlook; if the system's acquisition approach differs substantially from the plan; or if achieving performance requirements necessitates major alterations in the system configuration.

Uncertainty is also present in elements that define a system's configuration. This is referred to as *system definition uncertainty.* Examples include uncertainties in the amount of software to develop, the extent that code from another system can be reused, the number of workstations to procure, or the delivered weight of an end-item (e.g., a satellite).

B. Model Domain

Figure 1 illustrates the interrelationships between cost estimation uncertainty, requirements uncertainty, and system definition uncertainty. The *n*-system configurations shown are in response to the effects of requirements uncertainty. The cost-schedule probability models described in this paper apply within the domain shown in figure 1. They provide probability-based estimates of a system's cost and schedule for a given system configuration. When requirements uncertainty necessitates defining an entirely new system configuration, new cost-schedule probability models must also be developed specific to that configuration.

When a joint cost-schedule probability model has been developed on a program, analyses can be conducted to support risk management decisions. These include:

Baselining Program Cost and Schedule Risk—For a given system configuration, acquisition strategy, and resource estimation approach baseline probability distributions of a program's cost and schedule can be developed. These distributions can be periodically updated as the program's uncertainties change across the acquisition milestones. Generating these distributions supports defining a program cost and schedule that simultaneously have a specified probability of not being exceeded. This distribution also provides program managers with an assessment of the likelihood of achieving a budgeted or proposed cost and schedule or cost for a given schedule.

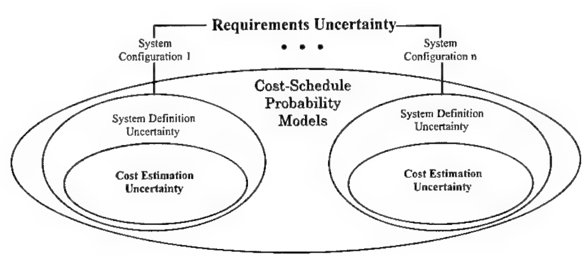

Figure 1. Types of Uncertainty Captured By Models

Estimating Reserves—An analytical basis for assesing cost reserve for a given schedule, or set of schedules, as a function of the uncertainties specific to a given system configuration can be developed. Sensitivity analyses can be conducted to assess how reserve levels are affected by changes in specific program risks. In addition, the relationship between the amount of reserve to recommend for a given level of confidence and a given schedule can be examined.

Conducting Risk Mitigation Tradeoff Analyses—Models can be developed to study the payoff of implementing specific risk mitigation activities (e.g., prototyping) on reducing cost and schedule variances. For instance, suppose the cost and schedule variances of a program are driven by uncertainty in the amount of new code to develop. Using these models, it can be determined whether investing in a prototyping effort markedly reduces this variance, and, therefore, lessen the cost and schedule reserves needed for the program.

III. BIVARIATE PROBABILITY MODELS FOR COST AND SCHEDULE

This section presents two bivariate probability models for quantifying uncertainty in cost and schedule estimates. Specifically, the bivariate lognormal and bivariate normal-lognormal models are described. They are candidate theoretical distributions that might be assumed by an analyst when estimates of joint or conditional cost-schedule probabilities are needed.

These models have two properties desirable for cost analysis applications. First, they directly incorporate correlation between cost and schedule on a given program. Second, their marginal distributions are either both lognormal or normal and lognormal. The latter property is reflective of distributions frequently observed in Monte Carlo simulations of program cost and schedule [4,5]. The following briefly describes each model. Throughout the remainder of this paper, the random variables X_1 and X_2 will denote program cost and schedule, respectively.

A. The Bivariate LogNormal Model

This section presents the bivariate lognormal model and summarizes its major characteristics. Monte Carlo simulations suggest that the lognormal distribution frequently approximates the sum of many positively correlated cost element random variables. Similarly, program schedule also tends toward lognormality if it is the sum of many positively correlated schedule activities. Thus, correlated lognormals often characterize a program's cost-schedule uncertainties. In cases where

- univariate lognormals approximate a program's cost and schedule distributions,
- circumstances (e.g., time, computing resources) preclude the development and implementation of formal Monte Carlo simulations,

then the bivariate lognormal can serve as an assumed, albeit non-unique, analytical model of the joint cost-schedule distribution.

A.I Model Definition

Suppose we have two random variables $Y_1 = \ln X_1$ and $Y_2 = \ln X_2$ where X_1 and X_2 are defined on $0 < x_1 < \infty$ and $0 < x_2 < \infty$. If Y_1 and Y_2 each have a normal distribution then, for $i = 1, 2$, the mean and variance of Y_i are

$$E(Y_i) = \mu_{Y_i} = \mu_i = \frac{1}{2}\ln\left[\frac{(\mu_{X_i})^4}{(\mu_{X_i})^2 + \sigma^2_{X_i}}\right] \tag{1}$$

$$Var(Y_i) = \sigma^2_{Y_i} = \sigma^2_i = \ln\left[\frac{(\mu_{X_i})^2 + \sigma^2_{X_i}}{(\mu_{X_i})^2}\right] \tag{2}$$

Assume that the pair

$$(X_1, X_2) \sim \text{Bivariate } LogN((\mu_1, \mu_2), (\sigma_1^2, \sigma_2^2, \rho_{1,2})) \tag{3}$$

is a bivariate lognormal distribution with density function

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} \dfrac{1}{(2\pi)\sigma_1\sigma_2\sqrt{1-\rho_{1,2}^2}\,x_1x_2} e^{-\frac{1}{2}w} \\ 0 \quad \text{otherwise} \end{cases} \tag{4}$$

where

$$w = \frac{1}{1-\rho_{1,2}^2}\left\{\left(\frac{\ln x_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho_{1,2}\left(\frac{\ln x_1 - \mu_1}{\sigma_1}\right)\left(\frac{\ln x_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\ln x_2 - \mu_2}{\sigma_2}\right)^2\right\}$$

and μ_i and σ_i^2 ($i = 1, 2$) are given by equations (1) and (2). The correlation term $\rho_{1,2}$ in equation (4) is

$$\rho_{1,2} = \frac{1}{\sigma_1 \sigma_2} \ln\left[1 + \rho_{X_1,X_2} \sqrt{e^{\sigma_1^2} - 1}\sqrt{e^{\sigma_2^2} - 1}\right] \quad (5)$$

A.II Model Characteristics

The marginal distributions of the bivariate lognormal distribution are

$$f_1(x_1) = \frac{1}{\sqrt{2\pi}\, \sigma_1 x_1} e^{-\frac{1}{2}\left[(\ln x_1 - \mu_1)^2 / \sigma_1^2\right]} \quad (6)$$

and

$$f_2(x_2) = \frac{1}{\sqrt{2\pi}\, \sigma_2 x_2} e^{-\frac{1}{2}\left[(\ln x_2 - \mu_2)^2 / \sigma_2^2\right]} \quad (7)$$

The conditional distributions are

$$X_1 | x_2 \sim LogN(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho_{1,2}(\ln x_2 - \mu_2),\, \sigma_1^2(1 - \rho_{1,2}^2)) \quad (8)$$

$$X_2 | x_1 \sim LogN(\mu_2 + \frac{\sigma_2}{\sigma_1}\rho_{1,2}(\ln x_1 - \mu_1),\, \sigma_2^2(1 - \rho_{1,2}^2)) \quad (9)$$

where

$$E(X_1 | x_2) = x_2^{\frac{\sigma_1}{\sigma_2}\rho_{1,2}} e^{\mu_1 - \frac{\sigma_1}{\sigma_2}\rho_{1,2}\mu_2 + \frac{1}{2}\sigma_1^2(1 - \rho_{1,2}^2)} \quad (10)$$

$$E(X_2 | x_1) = x_1^{\frac{\sigma_2}{\sigma_1}\rho_{1,2}} e^{\mu_2 - \frac{\sigma_2}{\sigma_1}\rho_{1,2}\mu_1 + \frac{1}{2}\sigma_2^2(1 - \rho_{1,2}^2)} \quad (11)$$

$$Var(X_1 | x_2) = x_2^{2\frac{\sigma_1}{\sigma_2}\rho_{1,2}} e^{2(\mu_1 - \frac{\sigma_1}{\sigma_2}\rho_{1,2}\mu_2)} e^{z^o}(e^{z^o} - 1) \quad (12)$$

$$Var(X_2 | x_1) = x_1^{2\frac{\sigma_2}{\sigma_1}\rho_{1,2}} e^{2(\mu_2 - \frac{\sigma_2}{\sigma_1}\rho_{1,2}\mu_1)} e^{z}(e^{z} - 1) \quad (13)$$

with

$$z^o = \sigma_1^2(1 - \rho_{1,2}^2), \quad z = \sigma_2^2(1 - \rho_{1,2}^2)$$

A.III An Illustration

This section illustrates an application of the bivariate lognormal distribution. Suppose the software cost and schedule of an Ada development task are needed. A simple Ada cost-schedule estimation model is presented in figure 2.

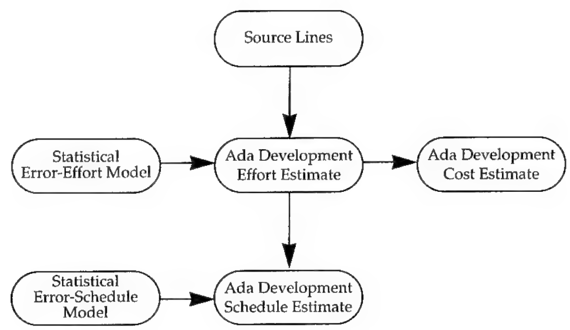

Figure 2. Ada Cost-Schedule Estimation Model

Suppose the uncertainty in the amount of Ada source lines to be developed is given as a triangular distribution, whose range of values is shown in figure 3. The Ada development effort (in staff months) is estimated according to

$$Effort \quad 8.2(Source\ Lines\ 1000)\varepsilon_{Effort} \quad (14)$$

where

$$\varepsilon_{Effort} \sim LogN(1.189, 0.277) \quad (15)$$

The development schedule (in months) is estimated according to

$$Schedule = 4.8(Effort)^{0.29}\,\varepsilon_{Sched} \quad (16)$$

where

$$\varepsilon_{Sched} \sim LogN(0.994, 0.0625) \quad (17)$$

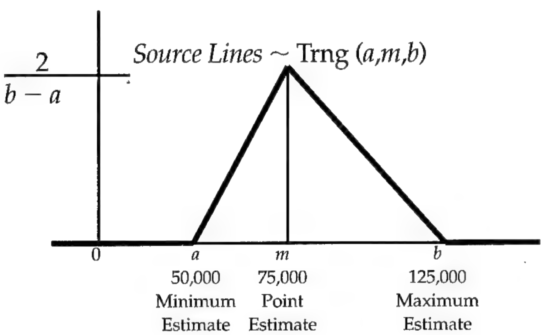

Figure 3. Ada Source Lines-Triangular Distribution

The development cost is estimated by

$$Cost = \lambda(Effort) \quad (18)$$

where λ = \$12,000 per staff month.

A Monte-Carlo simulation was performed for the model defined by equations 14-18. Summary cost-schedule statistics generated from that simulation are provided in table 2.

Table 2. Software Cost and Schedule Statistics

i	X_i	μ_{X_i}	$\sigma^2_{X_i}$	σ_{X_i}
1	*Cost* (\$M)	9.8	24.01	4.9
2	*Schedule* (Mos)	32.6	90.25	9.5
	Correlation ρ_{X_1,X_2}		0.46	

A comparison of the simulated marginal distributions of X_1 and X_2 with assumed theoretical lognormals is presented in figure 4. Observe how closely the simulated distributions match the hypothesized lognormal distributions. The simulation provides empirical evidence that the Ada cost and schedule distributions, in this illustration, can be characterized by correlated univariate lognormals. From this result, it is reasonable to assume that the bivariate lognormal is representative of the joint cost-schedule distribution. However, such an assumption does not guarantee it is the unique distribution. The true joint distribution of (X_1,X_2) cannot be uniquely determined from the marginal distributions of X_1 and X_2, unless they are uncorrelated. From the data in table 2, the parameters needed to specify a bivariate lognormal model can be computed. These data are summarized in table 3.

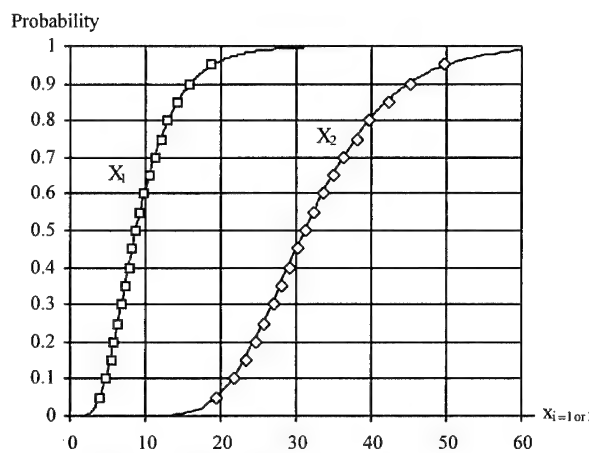

Figure 4. Simulated versus Theoretical Marginal Distributions[1]

Table 3. Joint Probability Model Parameter Set

i	μ_i	σ^2_i	σ_i
Bivariate LogNormal Model			
1	2.171 *(Eqt 1)*	0.223 *(Eqt 2)*	0.472
2	3.444 *(Eqt 1)*	0.082 *(Eqt 2)*	0.286
	Correlation $\rho_{1,2}$	0.48 *(Eqt 5)*	

Once the joint distribution of (X_1,X_2) has been specified, joint cost-schedule probabilities can be computed. For instance, suppose (X_1,X_2) is assumed to be bivariate lognormal with parameters given in table 3. Then the probability that the Ada development will cost between 7 and 14 million dollars (\$M) and be completed between 24 and 36 months (mos)

[1]The simulated marginal distributions are shown by the squares. The assumed marginals, theoretical lognormal distributions, are shown by the solid lines.

$$P(\$7\text{M} \le X_1 \le \$14\text{M and } 24 \text{ mos} \le X_2 \le 36 \text{ mos})$$

is $$\int_{24}^{36}\int_{7}^{14} f_{X_1,X_2}(x_1,x_2)\,dx_1 dx_2 = 0.29$$

In this case, the integrand is defined by equation 4. Joint cost-schedule probabilities for other regions of interest are determined in a similar manner.

Cost analyses must often focus on assessing the impact that a set of schedule outcomes has on the likelihood that an estimated program cost will not be exceeded. To make such assessments conditional distributions are needed. For the Ada cost-schedule model, the conditional cost distribution given a schedule outcome x_2 is

$$X_1|x_2 \sim \text{LogN}(2.171 + 0.792(\ln x_2 - 3.444), 0.172) \quad (19)$$

Table 4 provides the mean and standard deviation of $X_1 | x_2$ given two values of x_2. From figure 5, the impact of a schedule outcome on the probability distribution of cost can be seen. As x_2 increases the cumulative conditional cost distributions become "lazier." The increased laziness reflects a growth of nearly 3 million dollars in $E(X_1 | x_2)$ if the development schedule is 3 years (36 months) instead of 2 years (24 months). Although the higher cost may be undesirable, it is accompanied by a 3 year schedule, which has a lower chance of overrun (refer to figure 4) than the less expensive but higher risk 2 year schedule.

Linkages such as these between cost uncertainty and schedule can be made through the use of conditional distributions. In the early stages of a program, conditional distributions provide decision-makers valuable insights into the likelihood of achieving cost and schedule goals.

Table 4. Statistics from the Conditional Distributions in Figure 5

Given x_2 (Months)	$E(X_1 \mid x_2)$ (\$M)	$\sigma(X_1 \mid x_2)$ (\$M)
24	7.7	3.3
36	10.7	4.6

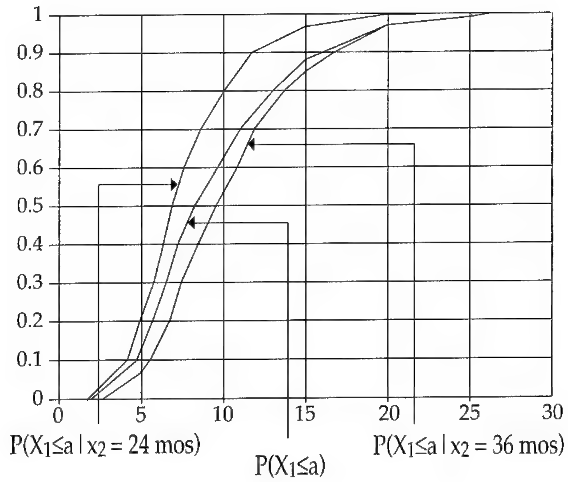

Figure 5. A Family of Conditional LogNormal Cost Distributions Ada Cost-Schedule Model Example

B. A Bivariate Normal-LogNormal Model

This section presents a bivariate normal-lognormal model and summarizes its major characteristics. An important feature of this model is its marginal distributions. One marginal is normal and the other is lognormal.

The previous section described circumstances that give rise to lognormal distributions for program cost and schedule. Normal distributions also occur frequently. For instance, when program

- cost is the sum of many uncorrelated WBS cost element random variables,
- schedule is the sum of many independent activities in a network,

then the normal often approximates the distribution of program cost or schedule.

In addition to the above instances, a normal distribution for program cost can arise from the way cost element distributions interact within a WBS. For example, when the following conditions exist

- the *PME* segment (refer to table 1) cost is normally distributed,
- the *PME* cost variance dominates a program's total cost variance,
- the *SS* segment (refer to table 1) is highly correlated to a normally distributed *PME* cost,

then program cost is approximately normal. Normality can also result when the *SS* segment is uncorrelated to a normally distributed *PME* cost.

In summary, when normal and lognormal distributions characterize either the cost or the schedule of a program, the bivariate normal-lognormal distribution can serve as an assumed joint probability model. This assumption, however, does not imply that the bivariate normal-lognormal is the unique distribution.

B.I Model Definition

Suppose we have two random variables $Y_1 = X_1$ and $Y_2 = lnX_2$ where X_1 and X_2 are defined on $-\infty < x_1 < \infty$ and $0 < x_2 < \infty$. If Y_1 and Y_2 each have a normal distribution then, for $i = 1, 2$, the mean and variance of Y_i are

$$E(Y_1) = \mu_{Y_1} = \mu_{X_1} = \mu_1 \tag{20}$$

$$Var(Y_1) = \sigma_{Y_1}^2 = \sigma_{X_1}^2 = \sigma_1^2 \tag{21}$$

$$E(Y_2) = \mu_{Y_2} = \mu_2 = \frac{1}{2}\ln\left[\frac{(\mu_{X_2})^4}{(\mu_{X_2})^2 + \sigma_{X_2}^2}\right] \tag{22}$$

$$Var(Y_2) = \sigma_{Y_2}^2 = \sigma_2^2 = \ln\left[\frac{(\mu_{X_2})^2 + \sigma_{X_2}^2}{(\mu_{X_2})^2}\right] \tag{23}$$

Assume that the pair

$$(X_1, X_2) \sim \text{Bivariate } N\,LogN((\mu_1, \mu_2), (\sigma_1^2, \sigma_2^2, \rho_{1,2})) \tag{24}$$

is a bivariate normal-lognormal distribution with density function

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} \dfrac{1}{(2\pi)\sigma_1\sigma_2\sqrt{1-\rho_{1,2}^2}\; x_2} e^{-\frac{1}{2}w} \\ 0 \quad \text{otherwise} \end{cases} \tag{25}$$

where

$$w = \frac{1}{1-\rho_{1,2}^2}\left\{\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho_{1,2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln x_2-\mu_2}{\sigma_2}\right) + \left(\frac{\ln x_2-\mu_2}{\sigma_2}\right)^2\right\}$$

and μ_i and σ_i^2 ($i = 1,2$) are given by equations (20) through (23). The correlation term $\rho_{1,2}$ in equation (25) is

$$\rho_{1,2} = \rho_{Y_1,Y_2} = \rho_{X_1, \ln X_2} = \rho_{X_1,X_2}\frac{(e^{\sigma_2^2} - 1)^{1/2}}{\sigma_2} \tag{26}$$

B.II Model Characteristics

A characteristic of the bivariate normal-lognormal distribution is that the distribution of X_1 is normal and the distribution of X_2 is lognormal. These marginal distributions are given by

$$f_1(x_1) = \frac{1}{\sqrt{2\pi}\,\sigma_1} e^{-\frac{1}{2}\left[(x_1-\mu_1)^2/\sigma_1^2\right]} \tag{27}$$

and

$$f_2(x_2) = \frac{1}{\sqrt{2\pi}\,\sigma_2 x_2} e^{-\frac{1}{2}\left[(\ln x_2-\mu_2)^2/\sigma_2^2\right]} \tag{28}$$

The conditional probability density function of X_1 given $X_2 = x_2$ is

$$X_1|x_2 \sim N(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho_{1,2}(\ln x_2 - \mu_2), \sigma_1^2(1-\rho_{1,2}^2)) \tag{29}$$

similarly

$$X_2|x_1 \sim LogN(\mu_2 + \frac{\sigma_2}{\sigma_1}\rho_{1,2}(x_1 - \mu_1), \sigma_2^2(1-\rho_{1,2}^2)) \tag{30}$$

The conditional means and variances are

$$E(X_1|x_2) = \mu_1 + \frac{\sigma_1}{\sigma_2}\rho_{1,2}(\ln x_2 - \mu_2) \tag{31}$$

$$E(X_2|x_1) = e^{\mu_2 + \frac{\sigma_2}{\sigma_1}\rho_{1,2}(x_1-\mu_1) + \frac{1}{2}\sigma_2^2(1-\rho_{1,2}^2)} \tag{32}$$

$$Var(X_1|x_2) = \sigma_1^2(1-\rho_{1,2}^2) \tag{33}$$

$$Var(X_2|x_1) = e^{2(\mu_2 + \frac{\sigma_2}{\sigma_1}\rho_{1,2}(x_1-\mu_1))} e^z(e^z - 1) \tag{34}$$

IV. Summary Comments

The joint probability models described in this paper provide an analytical basis for computing joint and conditional cost-schedule probabilities. These models are particularly useful when (i) quick probability-based assessments of achieving program cost and schedule are

needed, (ii) the work breakdown structure is used as the primary cost-schedule estimation method, and (iii) time or computing resources preclude the use of Monte Carlo simulation.

Selection of a particular model is guided by the marginal distributions it produces. For example, if the individual distributions of X_1 and X_2 are normal and lognormal, then the bivariate normal-lognormal model might be chosen for the joint distribution of the pair (X_1,X_2). This is because the bivariate normal-lognormal model produces normal and lognormal marginal distributions. However, it must be viewed that choosing this particular model makes an assumption about how the pair of random variables (X_1,X_2) is jointly distributed. Mentioned previously, the true joint distribution of (X_1,X_2) cannot be uniquely determined from only the individual distributions of X_1 and X_2.

A parameter required by the models in this paper is the correlation between program cost and schedule. This is a difficult parameter to determine. Ideally, it should be derived from values sampled by a Monte Carlo simulation of the cost-schedule relationships established in a program's work breakdown structure. Subjective assessments can be used in the absence of a simulation.

An important consideration regarding these models is that they do not reflect the causal impact that schedule compression or extension has on cost. Cost and schedule are treated as correlated random variables whose range of values are reflected by their marginal distributions. These ranges result from quantifying the uncertainties associated with cost estimation uncertainty and system definition uncertainty. Unrealistically compressing or extending schedule (due to missed milestones or program replans) can incur increased cost. In these circumstances a reassessment of the system's cost-schedule risk is warranted.

In an environment of limited funds and increasingly challenging schedules, it is incumbent upon analysts to continually examine affordability concerns relative to the likelihood of jointly meeting cost and schedule and cost for a given feasible schedule against specific tradeoffs in system requirements, acquisition strategies, and post-development support. Enabling options to be explored that offer decision-makers economically sound and risk mitigating choices throughout the life of a program is the cost analyst's aim and opportunity. Models and methodologies are tools that provide a means to that end.

ACKNOWLEDGMENTS

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Dynamical Instability in Combat Models: Computer Arithmetic and Mathematical Models of Attrition and Reinforcement

Dr. Julian I. Palmore
University of Illinois
j-palmore@cecer.army.mil

Application Areas:
Simulation of Combat Operations

OR Methodology:
Simulation, Advanced Computing, Deterministic Nonlinear Effects

INTRODUCTION

This paper considers two contributing causes of instabilities in combat models and complex simulations. These are (1) computer arithmetic and (2) nonlinear effects, including chaos. The paradigm of dynamics and dynamical systems sets this discussion. A reason for this is that many computations are performed algorithmically. When a computation requires algorithmic iteration, the algorithm functions as a dynamical system which may exhibit computational chaos. Complex models are decision making models that are designed using two or more different interacting modeling paradigms. For this reason combat models are complex models. The design of combat models is based on at least two paradigms: *discrete event simulation* and algorithmic calculations using *computer arithmetic*. Vagaries of computer arithmetic cause phase changes and, therefore, timing problems, in computational intensive discrete event simulations of decision making models. Nonlinear effects cause chaos and structural variance in the time evolution of complex models.

Discussions of computer arithmetic are focused on instabilities inherited from real arithmetic. We give a real world example of propagation of timing errors that are a direct effect of computer arithmetic.

Nonlinear effects are explored easily in simulations of simple models of attrition and reinforcement. We give a several parameter family of models of attrition and reinforcement. We analyze this family for chaos, strange attractors, and repellers of the dynamics.

COMPUTER ARITHMETIC EFFECTS

We give below a real world example of computer arithmetic effects and their tragic consequences. This demonstrates the fact that arithmetic effects can have significant consequences. For simulations these effects need not be limited to a statistically insignificant (less than 5%) range. Predicting the extent of divergence introduced by vagaries of computer arithmetic in complex simulations is difficult because of the enormous numbers of computational paths. In the Patriot example we see there is only one computational path of significance.

A DETAILED ANALYSIS OF THE PATRIOT EXAMPLE

A timing problem is illustrated by a significant real world example. An attack by a SCUD missile on Dhahran, Saudi Arabia, during the Persian Gulf war, was not engaged. As documented in a U.S. General Accounting Office report, an accumulation of timing error caused the failure of a radar system to detect and to identify a missile that should have been defended against by the PATRIOT [U.S. GAO, 1992]. This, in turn, resulted in a failure to launch a PATRIOT. The timing problem was a mismatch between a timing computation and the stroboscopic view by the radar. Time was accumulated as integer numbers of tenths of a second. Thus, 100 tenths equals 10 seconds. However, the arithmetic was done in binary, not in decimal. If finite precision is used, there is always a rounding error in the binary representation of 1/10. The timing error that results from the truncated expansion of 1/10 is directly proportional to the integer number of tenths. When the system operates for more than a few hours this error is appreciable. In this particular case, the system operated for 100 hours or 3,600,000 tenths of a second. Computer arithmetic with 24 bits was used for floating point calculations. A 24 bit word contains a sign bit, an exponent and a significand (also called mantissa). Precision refers to the number of bits in a significand. Precision 20 results in an error of about 2^{-23} in the binary representation of 1/10. One tenth is written as a binary expansion 0.000**11001100110011001100** | 1100... where truncation error is about 2^{-23} at precision 20. The difference between 1/10 and the expansion truncated at precision 20 is exactly 0.8×2^{-23}. The floating point calculation is in error by $3{,}600{,}000 \times 0.8 \times 2^{-23}$, (= 0.343322..), about 1/3 second. This error caused a shift of about 700 meters in the range gate (= 1/3 sec x 2000 m/s, the terminal velocity of SCUD) that was sufficient for the radar detection logic to fail to identify and flag the incoming SCUD as a missile that should be engaged [Palmore, 1992a]. See table on page 49.

COMPUTER ARITHMETIC

Computer arithmetic is the implementation in hardware of finite precision real arithmetic. Real arithmetic consists of the

field of real numbers with the binary operations of $(+, \times)$. The field axioms are associativity and commutativity in addition and multiplication, distributivity with respect to addition and multiplication, existence of additive and multiplicative inverses, and identities for addition and multiplication. *Multiplication is iterative addition of two real numbers* and *division is iterative subtraction of a nonzero real number from a real number.* These operations need not terminate in finitely many steps for arbitrary computable real numbers because arbitrary computable reals have infinite digit strings. Instabilities of real arithmetic are inherited by finite precision computer arithmetic.

MODELS OF COMPUTER ARITHMETIC

It is vital to recognize the importance of computer arithmetic and its vagaries in the execution of simulations of complex models where unstable behavior in embedded dynamical systems, even the arithmetic itself, can cause excessive divergences in results [Palmore, 1992c, 1991a,b; Palmore and Herring, 1990].

Models of computer arithmetic include algorithms that are used to implement the operations of addition, multiplication, subtraction, and division. These latter algorithms are implemented usually in hardware within the numeric coprocessor. Although the fundamental elements of shift and add are employed, there is a choice in the algorithm used for division. Models of computer arithmetic depend upon base of arithmetic, data types, precisions, roundings, and specifications.

The IEEE standards for binary floating point arithmetic and radix independent floating point arithmetic allow 12 combinations of precisions and roundings [ANSI/IEEE 1985]. IEEE precisions are 24, 53, and 64 binary bits. These are single, double and extended precisions, respectively. IEEE roundings are round to nearest, and three directed roundings: truncate (chop or round toward 0), round up (toward $+\infty$), round down (toward $-\infty$).

Each combination of precision and rounding is a different model of computer arithmetic for the computation of numerical quantities. These 12 combinations of precision and rounding allow a "poor man's" test of sensitivity. By running a computer simulation using different combinations of precision and rounding, a designer can test its sensitivity to computer arithmetic.

These models of computer arithmetic make a difference. For example, when using a pseudo random number generator of the form $x \to a \cdot x \bmod m$ (a PRIME MODULUS MULTIPLICATIVE LINEAR CONGRUENTIAL GENERATOR) with $a = 7^5$ (= 16807) and $m = 2^{31} - 1$ (= 2147483647) at least 46 bits of precision are needed to contain intermediate numerical results. Consistent answers cannot be obtained if this requirement is not met. The reason for this is that when rounding occurs in the integer arithmetic (in the floating point environment where less than 46 bits are allowed) the multiplying factor of 16807 causes immediate divergence between the exact PRNG sequence and the computed sequence [Palmore, 1992b, 1991a,b].

Consider the Simscript II.5 pseudo random number generator. The multiplier a = 630360016 and prime $m = 2^{31} - 1$ are the parameters [Fishman and Moore, 1986]. Intermediate products of multiplication exceed double precision. The code uses partial products

$$(A \cdot 2^N + B) \cdot (C \cdot 2^N + D) = A \cdot C \cdot 2^{2N} + (A \cdot D + B \cdot C) \cdot 2^N + B \cdot D.$$

The Simscript II.5 generator fits existing double precision by computing and retaining partial products $A \cdot C$, $A \cdot D$, $B \cdot C$, and $B \cdot D$. Special care has been taken to implement this simple PRNG so that there is 1 - 1 correspondence between the discrete arithmetic model and its computer implementation.

INSTABILITY OF MULTIPLICATION AND DIVISION

As every school child knows, *multiplication and division are hard to do correctly by hand because once a mistake is made the rest of the answer is wrong.*

Obviously, a division computation diverges rapidly once a mistake is made. Division is unstable because division is equivalent to a Bernoulli shift, a known chaotic operation [Palmore and McCauley, 1987; Palmore, 1988; Palmore and Herring, 1990].

EFFECTS OF TINY ROUNDINGS ON SIMULATIONS

If computer simulations are computational intensive, then the computations, performed typically by algorithms as iterative operations, can inherit instabilities from computer arithmetic. The important point is this. The reason algorithms perform calculations efficiently is that a lot of information is gained at each step. For division at least one digit is gained at every step. Very efficient division algorithms, such as Newton's method, allow a geometrically increasing number of digits to be gained step by step. Chaotic processes are being used for this purpose. If small errors are found in an unstable calculation, then a rapid divergence of solutions is guaranteed.

THERE IS A CURE FOR THE PROPAGATION OF ERRORS

A cure for the propagation of errors is to use indefinite precision calculations so that small errors remain small. It is when a calculation hits a boundary that an effect arises that is similar to turbulence in fluids. In this case finite precision establishes a "boundary layer" in which turbulence is produced and extends to chaos in the "bulk calculation."

When indefinite precision is used, the number of bits in a calculation is allowed to grow to a much larger number than standard precisions allow. Bulk effects are seen only when a boundary is reached and back propagation occurs. This approach is effective when an attempt is made to hold accuracy to a few digits uniformly throughout a calculation. Thus, one ensures enough room to work so that the number of steps available for errors to propagate far enough in precision to reach a boundary, to be reflected, and to propagate back to affect the accuracy desired is greater than the number of steps required to complete a calculation.

AVAILABILITY OF INDEFINITE PRECISION ARITHMETIC FOR EXACT COMPUTATIONS

Indefinite precision is not available in high level discrete event simulation languages. Indefinite precision has to be built especially for particular calculations. It is part of the design of algorithms and their implementations on machines.

There are computational programs that do indefinite precision arithmetic. This includes rational arithmetic where answers are retained in rational type. These programs give exact answers.

INFORMATION LOSS IN COMPUTER ARITHMETIC

Information is lost in finite precision arithmetic. Consider the set of significands in precision N that represent numbers in the interval $1 \leq x < 2$. What is the subset of significands of numbers such that the reciprocal of each, expanded in base 2, has period not exceeding N? Each rational number has a binary expansion that is eventually periodic. Most of the binary expansions of reciprocals have periods that exceed N. Thus, finite precision does not hold the information necessary to represent the period of the binary expansions of most numbers. These defects may appear to be highly technical in origin. They are fundamental in the sense that changes in precision and rounding do not change their effects.

EFFECTS OF COMPUTER ARITHMETIC ON ITERATIVE PROCESSES

Problems with computer arithmetic can arise in iterative processes such as discrete dynamical systems. An example is given by the difference equations that are written down from the continuous system of Lanchestrian differential equations. These differential equations have hyperbolas as solution curves. A significant point is to model the reinforcement strategy easily. As in every discrete dynamical system in which the model is a set of difference equations to be iterated with uniform step size, the time is

measured by the number of iterations performed. In particular, roundings and precisions will affect outcomes as iteration proceeds. Here is the main question. When will the effects of computer arithmetic be observed? The results of iterating the discrete system can be compared directly with evaluations of the analytic solution. When an attractor of the dynamics exists the only effect likely to be observed is a change in phase of outcomes as the model of arithmetic is changed. With limited numbers of reinforcement blocks, there may be too little time (that is, numbers of steps) for computer arithmetic effects to accumulate. For chaotic behavior, with unlimited numbers of reinforcement blocks and unlimited numbers of iterations, it is a question of how long it takes before significant phase changes between the outcomes occur using two different models of arithmetic. One measure of effect is changing the win-loss picture. How many steps will this take? [Palmore, 1991a,c]

MATHEMATICAL ANALYSIS OF MODELS OF ATTRITION AND REINFORCEMENT

This section is divided into two parts. The first part is about chaos; the second part is about nonmonotonicity of battle outcomes. The interest in chaos and nonmonotonicity of simulated battle outcomes in simple models of attrition and reinforcement has come primarily from the work of Dewar, Gillogly, and Juncosa at RAND (Dewar, et.al., 1991).

A mathematical analysis of a generic model of attrition and reinforcement by the author below demonstrates chaos as a consequence of a positive Lyapunov exponent and disorder given by moduli operations that simulate reinforcement (Palmore, 1992a). Unpredictability in actual battle outcomes is suggested by Singleton's work (Palmore, 1992a).

We show how nonmonotonicity in battle outcomes is obtained for free by the addition of "area fire" terms to linear Lanchester equations. We prove that chaos exists both in the linear discrete Lanchester equations with reinforcement and in the nonlinear discrete Lanchester equations with reinforcement.

CHAOS IN MODELS OF ATTRITION AND REINFORCEMENT

We consider the following system of first order linear differential equations of Lanchester,

$$dx/dt = -ay, \; dy/dt = -bx \tag{1}$$

where $x, y > 0$, and auxiliary conditions, a) if $y/x \geq r$, then x is replaced by $x + c$, repeatedly $N \geq 1$ times, until $y/(x+Nc) < r$ holds, b) if $y/x \leq s$, then y is replaced by $y + d$, repeatedly $K \geq 1$ times, until $(y+Kd)/x > s$ holds, where $a,b,c,d,r,s > 0$, and $r > s > \sqrt{(b/a)}$.

This model has area preserving flow because the divergence of the right hand side equals 0. It has a first integral $I(x,y) = ay^2 - bx^2$. The solution curves of the differential equations are hyperbolas and the solutions of the system of equations together with the auxiliary conditions are arcs of hyperbolas and isolated points.

A DISCRETE MODEL DERIVED FROM THE LINEAR MODEL

By writing down a set of first order linear difference equations to replace the system of first order linear differential equations (1) above, we obtain a discrete time dynamical systems model of attrition and reinforcement. The system of linear difference equations is,

$$x_{n+1} = x_n - hay_n, \; y_{n+1} = y_n - hbx_n \tag{2}$$

where $x_n, y_n > 0$, for all $n \geq 0$, $h > 0$ is a parameter, and auxiliary conditions, a) if $y_n/x_n \geq r$, then x_n is replaced by $x_n + c$, repeatedly $N \geq 1$ times, until $y_n/(x_n+Nc) < r$ holds, and b) if $y_n/x_n \leq s$, then y_n is replaced by $y_n + d$, repeatedly $K \geq 1$ times, until $(y_n+Kd)/x_n > s$ holds, where a,b,c,d,r,s, and $h > 0$, and $r > s > \sqrt{(b/a)}$.

For $h \neq 0$, the flow of the discrete model is *area decreasing* (the determinant of the mapping equals $1 - h^2ab < 1$.) We consider only those values of h for which the determinant is positive, i.e. $1 - h^2ab > 0$. An attractor of the discrete flow is a compact, invariant set. Therefore, any attractor has measure 0.

Lyapunov multipliers and Lyapunov exponents are computed easily. The multipliers are

given by $\Lambda_{\pm}(h) = 1 \pm h\sqrt{(ab)}$. Lyapunov exponents are $\lambda_{\pm}(h) = ln\,[1 \pm h\sqrt{(ab)}]$. By scaling arguments for piecewise linear maps, this allows us to compute the Hausdorff dimension of the attractor A(h) as

$$HD[A(h)] = 1+\lambda_{+}(h)/\,|\lambda_{-}(h)|$$

for any $h > 0$. For $h \geq 0$, we have $HD[A(h)] = 2 - h\sqrt{(ab)} + O(h^2ab)$. Thus, the Hausdorff dimension of A(h) is less than 2 for $h > 0$ so that A is a fractal attractor.

We have proved the existence of a fractal attractor, a strange attractor, of the discrete time dynamics for every $h > 0$ such that the determinant of the mapping is positive.

A first integral for this discrete flow is given by $I(x_{n+1}, y_{n+1}; x_n, y_n) = (ay_{n+1}^2 - bx_{n+1}^2)/(ay_n^2 - bx_n^2) = 1 - h^2ab$. This expression is independent of the orbit. The integral $I(x_{n+1}, y_{n+1}; x_n, y_n)$ is called an isoenergetic first integral and it depends only upon h, a, and b.

It is clear that chaos exists in this dynamical system. There is a strange attractor and the system has a positive Lyapunov exponent for every $h > 0$ that we consider. This suffices for chaos (Palmore, 1991a). The positive Lyapunov exponent ensures a sensitive dependence on initial conditions for this discrete dynamical system. Chaos exists for all choices of h, a, and b such that $1-h^2ab > 0$. In particular, the Hausdorff dimension of the attractor, HD[A], can be computed easily as a function of the parameters h, a, and b.

The cases considered by Dewar, Gillogly and Juncosa of RAND are subsumed by the family of discrete dynamical systems above (Dewar, et. al., 1991). In RAND's simple model of attrition and reinforcement the discrete dynamical system is piecewise linear with a constant derivative. From our analysis above we know immediately several properties of the dynamics. We know the Lyapunov multipliers, a positive Lyapunov exponent, the directions of expansion and contraction, the measure and dimension of the attractor. The dynamical system is chaotic by a positive Lyapunov exponent. This simple model of attrition and reinforcement is an example of predator-predator interaction with restocking of each predator. Combat is distinguished from other model types: predator-prey, competing species, and logistic.

NONMONOTONICITY AND NONLINEARITY IN MODELS OF ATTRITION AND REINFORCEMENT

We add to the model of linear differential equations the terms for "area fire" used by Lanchester. The differential equations are written,

$$dx/dt = -Ay - ACxy, \quad dy/dt = -Bx - BDxy \tag{3}$$

where $x, y > 0$, and auxiliary conditions, a) if $y/x \geq r$, then x is replaced by $x + c$, repeatedly $N \geq 1$ times, until $y/(x+Nc) < r$ holds, and b) if $y/x \leq s$, then y is replaced by $y + d$, repeatedly $K \geq 1$ times, until $(y+Kd)/x > s$ holds, where $A,B,C,D,c,d,r,s > 0$, and $r > s > \sqrt{(B/A)}$.

We remark that the criterion $\sqrt{(B/A)}$ follows from the an analysis of the integral curve as it approaches the origin. This model has area decreasing flow (the divergence of the right hand side equals $-ACy - BDx < 0$ for all $x, y > 0$. This implies that any attractor of the dynamics has measure zero. Thus, a strange attractor exists when the flow is wrapped around a bounded region indefinitely by using unlimited reinforcements.

A FIRST INTEGRAL (CONSTANT OF MOTION)

A first integral (constant of motion) for the system of linear differential equations is $I_0(x,y) = ay^2 - bx^2$. The curve defined by $I_0(x,y)=C$ (constant) is an orbit of a solution of the differential equation. The orbit through (0,0) is a straight line $y = \sqrt{(b/a)}\,x$, $x \geq 0$. By integrating equations (3) we find an analytic first integral I(x,y) for the nonlinear differential equations,

$$I(x,y) = (B/C)x - (A/D)y - (B/C^2)ln(1+Cx) + (A/D^2)ln(1+Dy).$$

If $Cx = Dy$ and $(B/C)x = (A/D)y$, then the orbit that passes through the origin is a straight line. In any event, setting $I(x,y) = 0$ and finding the orbit for $x, y > 0$ yields a curve that separates the quadrant into two regions where a win occurs without reinforcement. In order to specify

the reinforcement criterion above, that $r > s > \sqrt{(B/A)}$, we examine the direction of approach to the origin (0,0) of the integral curve defined by $I(x,y) = 0$ for $x, y > 0$. We want to find the tangent direction of the curve at (0,0) that is the limiting direction of the vector field (3) as the curve approaches (0,0). The tangent direction is,

$$dy/dx = Bx(1+Dy)/[Ay(1+Cx)]. \tag{4}$$

A simple procedure yields the limiting direction along the curve defined by $I(x,y) = 0$. We set $y = \alpha x$ and substitute into (4). This yields at $x = 0$ the expression $B/A = \alpha^2$. With limited reinforcements, nonmonotonicity of battle outcomes is possible in this model. With unlimited reinforcements chaos occurs.

A DISCRETE MODEL DERIVED FROM THE NONLINEAR MODEL

By using the same technique to write down a replacement of the derivative as a finite difference, we obtain a discrete dynamical system for the nonlinear differential equations (3).

$$\begin{aligned} x_{n+1} &= x_n - hAy_n(1+Cx_n), \\ y_{n+1} &= y_n - hBx_n(1+Dy_n) \end{aligned} \tag{5}$$

This system of difference equations is nonlinear in x_n and y_n. By adding criteria for reinforcements the system of difference equations and auxiliary conditions is nonlinear in two distinct ways. The reinforcement strategy causes wraparound, a nonlinear effect.

CHAOS AND NONMONOTONICITY

We define a mapping F in two dimensions by,

$$F: (x, y) \rightarrow (x - hAy[1+Cx], y - hBx[1+Dy]) \tag{6}$$

By differentiating (6) we obtain the Jacobian matrix DF(x,y). Finally, by computing the determinant $|DF(x, y)|$ we obtain the Jacobian J(F) of the mapping F.

$$\begin{aligned} J(F) = 1-h^2AB - (hBD+h^2ABC)x \\ - (hAC+h^2ABD)y. \end{aligned} \tag{7}$$

For $x, y > 0$, $J(F) < 1$. This shows that the nonlinear "area fire" terms in the difference equations (5) only increase the rate of area decrease for $h > 0$. Thus, for all cases in which $h, x, y > 0$, the measure of an attractor equals 0.

We show by using (7) that a positive Lyapunov exponent exists for bounded orbits of the nonlinear difference equations (5). Let $\Lambda_{\pm}(x,y)$ denote eigenvalues of the derivative matrix of (6). We compute the multipliers from

$$\Lambda^2 - [2 - h(ACy+BDx)]\Lambda+[1 - h(ACy+BDx) - h^2AB(1+Cx+Dy)]=0.$$

For $A, B, C, D, h, x, y > 0$, we obtain for Λ the expression,

$$\Lambda = 1 + ½h(ACy+BDx)[-1 \pm \sqrt{(1 + [4AB(1+Cx+Dy)/(ACy+BDx)^2])}].$$

This demonstrates that $\Lambda_+ > 1$ and $_\Lambda < 1$ for all choices of $A, B, C, D, h, x, y > 0$. For $C=D=0$ we recover from the quadratic equation, $\Lambda_{\pm} = 1 \pm h\sqrt{(AB)}$, Lyapunov multipliers of the linear system.

This demonstrates a positive Lyapunov exponent for every orbit of the system (5) that is bounded away from (0,0). This implies chaos exists in the discrete dynamical system (5).

Fig. 1 is a 3-dimensional Battle Outcome Diagram for initial strengths of Red (fixed per panel) and Blue (increasing along horizontal). Fig.1 shows nonmonotonicity as the effect of nonlinearity as the strengths of area fire terms are increased from zero to positive in 0.01 increments of the coefficients. A nonlinearity effect is expected. As the initial strength of Blue force is increased, we expect area fire to be significant. This is illustrated in Fig. 1 for fixed initial strength of Red. We follow Blue's initial strength, moving from left to right along a horizontal line. At first Red wins. As Blue's initial strength increases, Blue wins. Eventually, area fire gives Red a significant advantage. As Blue's strength increases further, Red wins.

CONCLUSIONS

Mathematical analysis has shown the existence of chaos for the system of linear difference equations with reinforcements and the existence

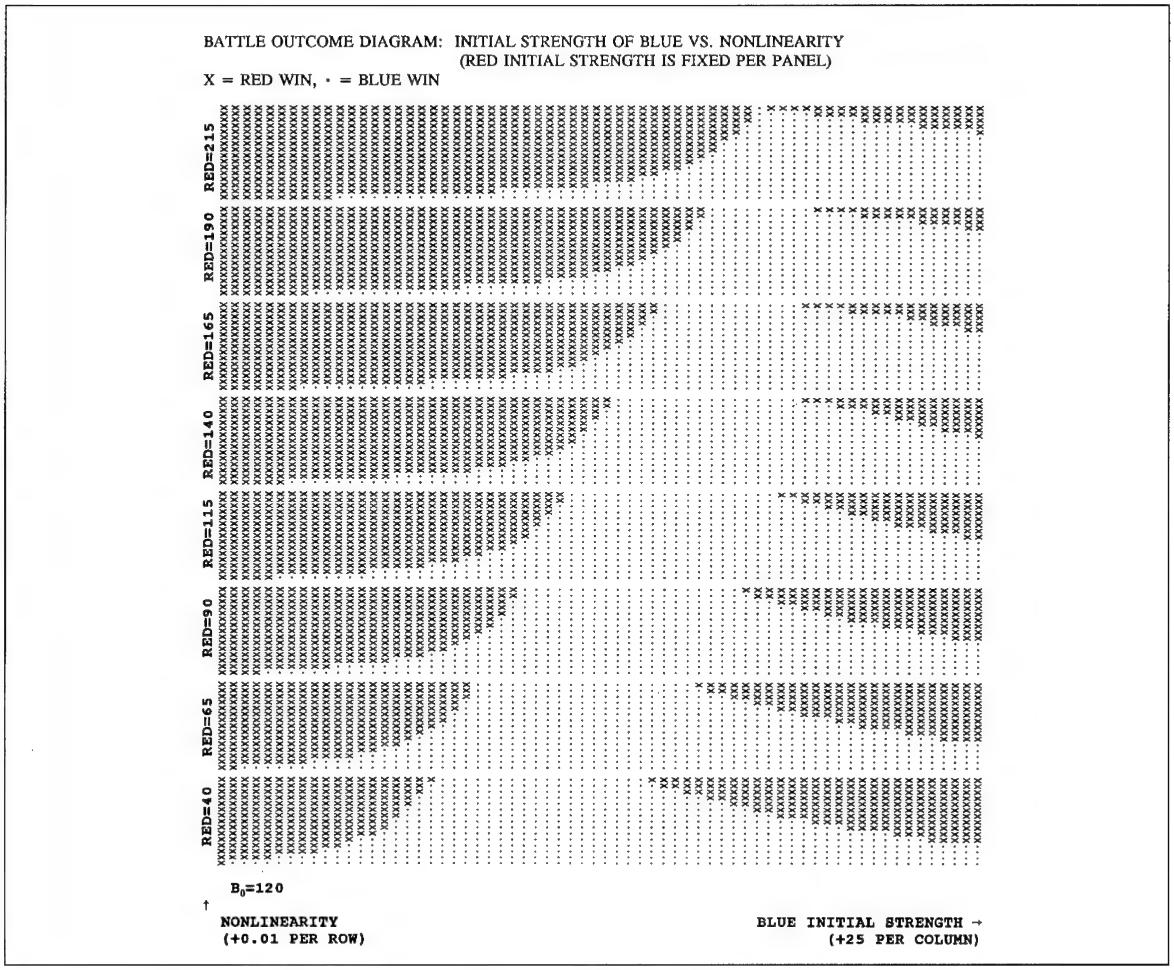

Figure 1: Battle Outcome Diagram: Initial Strength of Blue vs. Nonlinearity

TABLE

HOURS	SECONDS†	CALCULATED TIME (SECONDS)	IN ACCURACY (SECONDS)	SHIFT (METERS)
0	0	0	.0	0
1	3600	3599.9966	.0034	7
8	28800	28799.9725	.0275	55
20	72000	71999.9313	.0687	137
48	172800	172799.8352	.1648	330
72	259200	259199.7528	.2472	494
100	360000	* 359999.6667	.3433	687

DATA FROM APPENDIX II

EFFECT OF EXTENDED RUN TIME ON PATRIOT OPERATION

GAO/IMTEC-92-26 Patriot Missile Software Problem

† This column is calculated exactly as 10 × Integer count of 1/10 seconds.

* The second digit **6** is a misprint in the report.

of nonmonotonicities for the system of nonlinear difference equations with moduli operations. Nonlinearities in the equations give rise to non-monotonicity and the reinforcements with positive Lyapunov exponent give rise to chaos.

ACKNOWLEDGEMENTS

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Military Training Resource Scheduling: System Model, Optimal and Heuristic Decision Processes

LTC Michael L. McGinnis
Operations Research Center
U.S. Military Academy
fm0768@exmail.usma.mil

Professor Emmanuel Fernández-Gaucherand
Systems and Industrial Engineering, The University of Arizona
emmanuel@sie.arizona.edu

Dr. Pitu B. Mirchandani
Systems and Industrial Engineering, The University of Arizona
pitu@sie.arizona.edu

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Application Areas:
Resource Scheduling, Decision Support System Development

OR Methodology
Dynamic Programming and Heuristic Methods

ABSTRACT

The United States Army trains thousands of new soldiers each year to fill vacancies in Army organizations. Initial entry training consists of two sequential phases: Basic Combat Training followed by Advanced Individual Training. Until recently, manual heuristic methods were used to schedule hundreds of training companies for initial entry training. Scheduling training companies primarily involves deciding how many trainees to assign to training companies each week. This paper formulates a mathematical dynamic model of the Basic Combat Training phase of initial entry training. We also present two approaches for scheduling training resources. One is a mathematical decision model for optimally scheduling training resources based on dynamic programming. The second is a fully automated heuristic procedure implemented in an operational decision support system (DSS) for managing the Army's resources for initial entry training. Computational experiments reveal that the heuristic procedure developed is indeed computationally efficient. Furthermore, the heuristic provides "good" solutions in terms of three performance measures: training quality as measured by an instructor-to-trainee ratio, resource utilization, and training costs.

1. INTRODUCTION

Each year the United States Army recruits and trains thousands of new soldiers. Responsibility for initial entry training belongs to the Unites States Army Training and Doctrine Command (TRADOC) headquartered at Fort Monroe, Virginia. The military installations responsible for training new soldiers are scattered across the United States as shown below in Figure 1.

Entry level training consists of two phases: Basic Combat Training (BCT), normally lasting 8 weeks, followed by Advanced Individual Training (AIT) that varies from 5 to 50 weeks. The variability in time of AIT reflects curriculum differences across the AIT programs administered at different installations. In One-Station-Unit Training (OSUT), BCT and AIT take place at the same installation. Figure 2 illustrates an *aggregated* view of the initial entry training process. In this case, the training companies from all installations may be viewed as a single group of *reusable* resources.

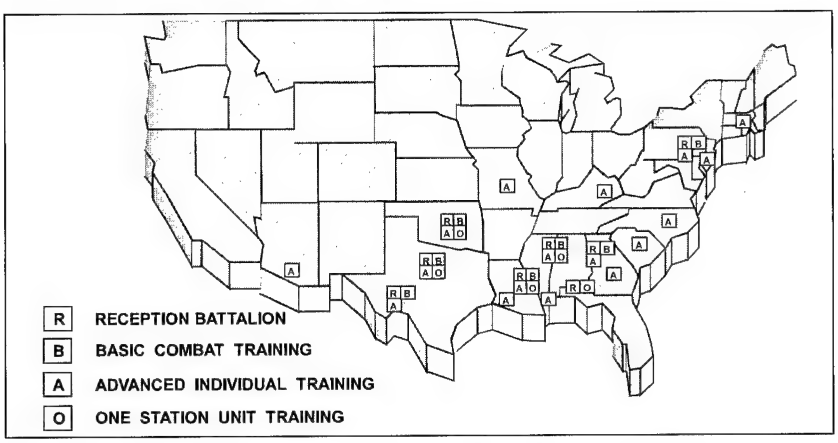

Figure 1: Initial entry training installations (1990).

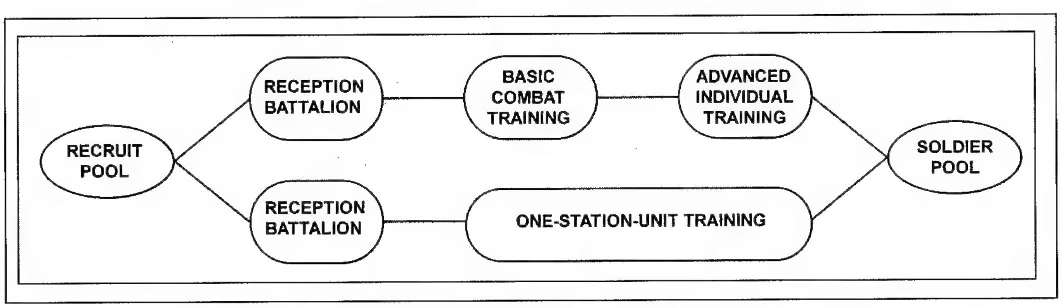

Figure 2: Aggregated view of the initial entry training process.

Proper management of the Army's initial entry training program demands timely scheduling of many reusable training resources. This paper deals with the complex, practical logistics problem of scheduling hundreds of basic training companies throughout the planning horizon for training new soldiers. Training company scheduling involves deciding how many trainees to assign to training companies each week and how many weeks the companies remain busy with training.

Until recently, training managers used manual and partially automated heuristic methods to schedule training companies for basic training. Severe shortcomings exist with these methods. First, assigning trainees to training companies and determining the number of weeks a training company remains busy training recruits were decided by trial-and-error. Second, it was possible for different analysts to generate different solutions for the same recruitment scenario. Third, no methods existed for conducting comparative analyses to evaluate the quality of competing feasible training schedules. Finally, the temporal interdependence of decisions makes decision variables in the future periods depend on current decision variables. This complicates resource scheduling and made the manual generation of week-by-week training schedules an extremely tedious, time-consuming task.

The major objectives of this paper are as follows. First, we formulate a mathematical dynamic model of the Basic Combat Training phase of initial entry training. Then we formulate a decision model for optimally scheduling training resources based on dynamic programming. Next, an improved, automated heuristic procedure for scheduling training resources is presented that we implemented in a decision support system (DSS). The heuristic procedure incorporates a measure of training "quality" into the objective function for comparing competing feasible training schedules. Computational experiments reveal that the heuristic procedure developed is computationally efficient and provides "good" scheduling solutions. Training schedule quality is determined by three performance measures: training quality as measured by an instructor-to-trainee ratio, resource utilization, and training costs.

Previous work somewhat related to our problem includes that of Yang and Ignizio (1987). Their problem involved scheduling training resources and training activities for a fixed number of US Army battalions located at the same training installation. Constraints existed on the type and quantity of training resources to be scheduled as well as precedence relationships between tasks and training units. These features of the problem occasionally required two or more battalions to work together and share resources in accomplishing some training tasks. The authors developed a two-phase heuristic program for scheduling training activities and resources.

The structure of the basic training scheduling problem (see Section 2) is slightly similar to the economic lot-sizing problem (ELSP) from manufacturing first described by Wagner and Whitin (1958) and Manne (1958). Notable solution methods and extensions to the problem discussed by Graves (1981), Blackburn and Millen (1982), Afentakis, Gavish, and Karmarkar (1984), Billington, McClain, and Thomas (1986), among others, suggested ideas for modeling and solving the basic training problem. In another related area, Rao (1990) developed a planning model that minimized fixed and variable recruitment and training costs for a personnel management system. Rao's approach motivated our development of a cost functional for the basic training scheduling problem.

2. MODEL FORMULATION

This section presents the mathematical model of the basic training problem. Practical aspects of Basic Combat Training essential to proper development of a dynamic training system model are discussed first.

Estimating the Weekly Arrival of New Trainees

For the version of the basic training problem presented here, we estimate trainee arrivals ahead of time by week t and year j for the entire planning horizon of T_J weeks. Recruiting continues throughout the year and focuses primarily on young people in their final year of high school. Force structure personnel requirements

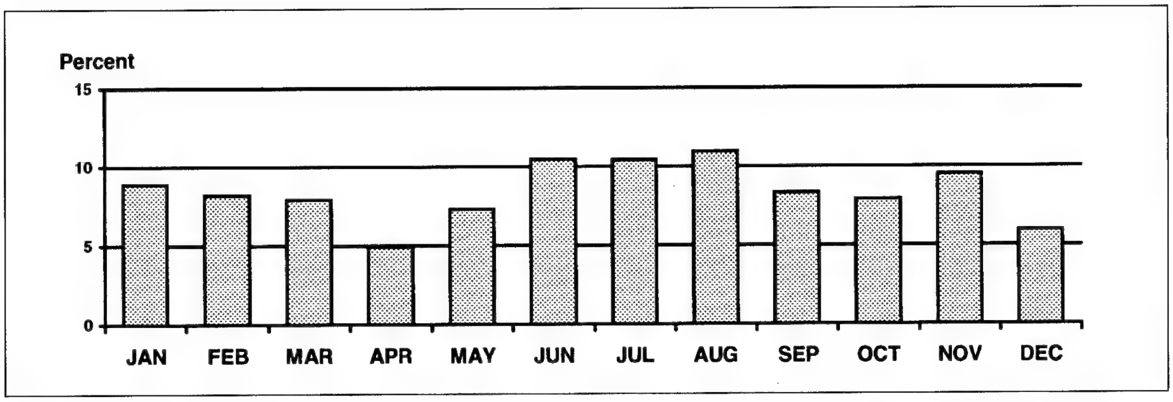

Figure 3: Percent of annual recruit arrivals by month.

may cause annual recruiting targets to vary from year-to-year. However, analysis of historical recruitment data reveals that the distribution of weekly trainee arrivals at all training installations throughout the year remains relatively stationary (see Figure 3). Weekly trainee arrivals are forecasted by applying the historical distribution of trainee arrivals to the annual recruiting targets for each year j of the planning horizon. The absence of a random disturbance makes this formulation of the basic training problem completely deterministic.

Dynamics of Varying Training Company Strength

Company strength, the number of trainees assigned per training company, is bounded below at 150 and above at 250 trainees. In practice, training managers assign all companies scheduled to start training in the same week equal strengths. This rule simplifies logistical problems relating to training program management and has been incorporated into the basic training model presented here.

Determining the company strength for a given week requires two critical elements of information. These are the number of trainees who report for training in week t and the number of training companies available at the beginning of week t to start training new soldiers. However, the number of training companies available to start training in week t depends upon past company strength decisions. In any week, it is possible for company strength decisions from previous weeks to cause a *training company shortfall.* A shortfall occurs when the number of training companies is not sufficient to handle the arrival of trainees, also referred to as *training load.*

Instructor-to-Trainee Ratio

Basic training is designed to be highly stressful for new trainees. Training managers rely on high quality instruction and close supervision of trainees to somewhat offset the negative effects of stress on learning. In practice, training managers attempt to keep the ratio of *instructors-to-trainees* around 1-to-16 or higher, if possible. This results in company strengths of approximately 200 trainees per company. This ratio is the primary performance measure for the optimal and automated heuristic scheduling methods presented here.

Compressing-the-Load

Each week as trainees report to basic training installations, they are assigned to training companies. This *fill week* runs from Saturday through midnight Thursday. Basic Combat Training begins on Friday and normally lasts 8 weeks. At the completion of basic training, training managers schedule a *maintenance week* for each training company before starting the next training cycle. This 10-week sequence is called a *normal training cycle.*

In some cases, it may not be possible to eliminate a training company shortfall by adjusting company strengths alone. Another way to correct the shortfall is to shorten the training cycle length for companies that started either 8 or 9 weeks earlier. This is done by eliminating either the fill week or the maintenance week, or both.

This practice, called *compressing-the-load,* shortens or eliminates the break between training cycles and is known to degrade training cadre effectiveness. Therefore, training managers only compress the load when the demand for training companies cannot be met by adjusting company strengths.

Mathematical Notation

- j: year of the planning horizon, $j \in \{1,2,\ldots,J\}$.
- t: week of a given year j, $t \in \{1,2,\ldots,T_j\}$ where T_j is the number of weeks in year j.
- $\delta(t)$: trainee show rate for week t where $0 \leq \delta(t) \leq 1$.
- $p(t)$: relative frequency distribution of trainee arrivals over week t of any year.
- D_j: number of training companies deactivated in year j.
- M_j: number of training companies available at the beginning of year j.
- R_j: recruiting objective for year j determined by Department of the Army.
- $d(t_j)$: number of training companies to deactivate in week t of year j.
- $r(t_j)$: estimated number of trainees that show up for training in week t of year j.
- $x(t_j)$: strength of companies starting in week t of year j.
- $\overline{X}$: upper bound for training company strength.
- $\underline{x}$: lower bound for training company strength.
- $y(t_j)$: training cycle length for companies starting in week t of year j.
- $\overline{Y}$: upper bound for training cycle length.
- $\underline{y}$: lower bound for training cycle length.
- $I(t_j)$: number of idle training companies at the beginning of week t of year j.
- $\overline{I}$: upper bound for the idle training company constraint.

Modeling Constraints

$\underline{x} \leq x(t_j) \leq \overline{X}$: company strength constraint. (1)

$\underline{y} \leq y(t_j) \leq \overline{Y}$: training cycle constraint. (2)

$M_j \geq D_j$: deactivation scenario constraint. (3)

$d(t_j) \geq 0 \;\forall\, (t,j)$: company deactivation constraint. (4)

$0 \leq I(t_j) \leq \overline{I} \;\forall\, (t,j)$: problem feasibility constraint. (5)

$r(t_j) = \delta(t)\, p(t)\, R_j$: expected number of trainees to arrive for training in week t of year j. The recruiting objective R_j, determined by the Department of the Army, is greater than the number of new soldiers required to meet the personnel needs of the Army in a given year j. (6)

3. OPTIMAL DECISIONS USING DYNAMIC PROGRAMMING

We simplified the dynamic programming (DP) formulation of the basic training problem by removing the training company deactivation decision $d(t_j)$. This was done by requiring these decisions to be made prior to implementation of the dynamic programming algorithm.

Stages

In the basic training problem, stages are denoted by week t and year j. The planning horizon consists of T_J discrete, identical weeks where $t \in \{1,2,\ldots,T_j\}$ and $j \in \{1,2,\ldots,J\}$.

Scheduling Decisions and Scheduling Policy

In any week $t \in \{1, 2,\ldots,T_J - 1\}$, company strength $x(t_j)$ and training cycle length $y(t_j)$ decisions for all training companies are made at the beginning of the week. We require that $x(t_j) \in \Omega$ and $y(t_j) \in \Lambda$. Ω and Λ are decision spaces consisting of bounded integer sets specified by the company strength constraint $\underline{x} \leq x(t_j) \leq \overline{X}$ and the training cycle constraint $\underline{y} \leq y(t_j) \leq \overline{Y}$, respectively. The subsets of feasible decisions to take at each stage t are denoted by $X[t_j, I(t_j)] \subset \Omega$ and $Y[t_j, I(t_j)] \subset \Lambda$. This notation indicates that decision elements belonging to these two subspaces depend upon both the stage t and the state $I(t_j)$ of the basic training system. A sequence of such decisions, denoted by π, is represented by

$$\pi = \begin{Bmatrix} x(1_1),\, x(2_1),\ldots,x(T_J - 1); \\ y(1_1),\, y(2_1),\ldots,y(T_J - 1) \end{Bmatrix}$$

The set of all feasible sequences Π consists of all solutions satisfying constraints (1) through (5) above.

Objective Function

Here we are interested in obtaining the optimal training schedule that maximizes the "quality" of training as measured by the ratio of instructors to trainees. In practice, we maximize the instructor-to-trainee ratio by minimizing company strengths. Coincidentally, this simultaneously minimizes idle training companies which serves as a measure of training resource utilization.

Assuming one instructor per training company for simplicity, then for each sequence of decisions $\pi \in \Pi$ there is a corresponding value $K\pi$ based on the instructor-to-trainee ratio that provides a measure of quality to be maximized. This measure is given by

$$K_\pi = \sum_{t_j=1_1}^{T_J-1} \frac{1}{x(t_j)} \tag{7}$$

The optimal sequence of decisions π^* is the one that maximizes the following objective function for a fixed initial state:

$$K_{\pi^*} = \max_{\pi \in \Pi} K_\pi \tag{8}$$

State Transition Equation

The state of the basic training system evolves according to the following balance equation for idle training companies:

$$I(t_j+1) = f\left[t_j, I(t_j), x(t_j)\right] = I_j(t) - \frac{r(t_j)}{x(t_j)} + \sum_{l \in L} \frac{r(t_j - l)}{x(t_j - l)} \tag{9}$$

$f\left[t_j, I(t_j), x(t_j)\right]$ is explicitly defined as an equivalent representation of the right hand side of (9). $\frac{r(t_j)}{x(t_j)}$ is an estimate of the number of companies to begin training in week t of year j. represents the number of companies that become available at the beginning of week t+1 to start another training cycle having just completed one that began either 8, 9, or 10 weeks earlier (see *Compressing-the-Load*). The possible values for $l \in L$ are contained within the set $L \in \{(10), (10,9), (10,9,8)\}$ (see McGinnis (1994), Section 2.3, for details).

To maintain the integer value of $I(t_j + 1)$ in (9), we round as follows:

For $x(t_j) < \overline{X}$: $(t_j+1) = I(t_j) - \left\lfloor \frac{r(t_j)}{x(t_j)} \right\rfloor + \left\lfloor \sum_{l \in L} \frac{r(t_j - l)}{x(t_j - l)} \right\rfloor$; (10)

For $x(t_j) = \overline{X}$: $I(t_j+1) = I(t_j) - \left\lceil \frac{r(t_j)}{x(t_j)} \right\rceil + \left\lceil \sum_{l \in L} \frac{r(t_j - l)}{x(t_j - l)} \right\rceil$.(11)

When the current company strength constraint is *tight*, that is, an equality constraint exists at the upper bound, then the fractional part of a training company is rounded up as denoted by the ceiling operator $\lceil * \rceil$. When all training companies are at full strength, the number of trainees represented by the fractional part of a training company can only begin basic training if an additional training company is scheduled to start. In all other cases, the fractional part may be dropped as denoted by the floor operator $\lfloor * \rfloor$. This rule is consistent with how training managers size training companies in practice when all training companies have not been filled to capacity. For simplicity, the floor and ceiling operators of (10) and (11), respectively, will not be repeated for every reference to training company computations. However, it is to be understood that these rules are in effect throughout the paper.

State Augmentation

Company strength $x(t_j)$ and cycle length $y(t_j)$ decisions depend upon past information that cannot be summarized in $I(t_j + 1)$ alone. State augmentation makes additional information needed for making scheduling decisions available in the current week (see Bertsekas). This requires incorporating additional variables into the formulation of the problem that can significantly increase the number of computations required to generate an optimal solution as well as the amount of computer memory required. Therefore, the state space is augmented by only the <u>minimum</u> number of variables necessary to make a decision in each week. For example, consider a simplified version of the problem where training cycle length is fixed at $y(t_j) = 10$. This eliminates training cycle length as a decision element but it also creates a 9-week time lag in information about previous $x(t_j)$ decisions needed to make company strength decisions in the current week. The minimally augmented state of the system for this case is given by (12).

$$\begin{bmatrix} I(t_j+1) \\ s^1(t_j+1) \\ s^2(t_j+1) \\ s^3(t_j+1) \\ s^4(t_j+1) \\ s^5(t_j+1) \\ s^6(t_j+1) \\ s^7(t_j+1) \\ s^8(t_j+1) \\ s^9(t_j+1) \end{bmatrix} = \begin{bmatrix} I(t_j) - \frac{r(t_j)}{x(t_j)} + \frac{r(t_j-9)}{s^1(t_j)} \\ s^2(t_j) = x(t_j-8) \\ s^3(t_j) = x(t_j-7) \\ s^4(t_j) = x(t_j-6) \\ s^5(t_j) = x(t_j-5) \\ s^6(t_j) = x(t_j-4) \\ s^7(t_j) = x(t_j-3) \\ s^8(t_j) = x(t_j-2) \\ s^9(t_j) = x(t_j-1) \\ x(t_j) \end{bmatrix} \tag{12}$$

One can think of $\{s^1(t_j), s^2(t_j),...,s^9(t_j)\}$ as "registers" for temporarily storing the required information as the system evolves (see Bertsekas). The sequence $\{I(t_j), s^1(t_j), s^2(t_j),...,s^9(t_j)\}$ constitutes the state of the system in our formulation.

Estimating the Size of the Problem

A combinatorial explosion of the state space occurs when attempting to obtain an optimal solution to the real-world basic training problem using exact methods such as complete enumeration or dynamic programming. The augmented state space for a single week of the basic training problem, based on 1990 training data, is estimated by enumeration of the following state variables.

- Maximum number of idle training companies $I(t_j + 1)$: 130.
- Total company strength values $x(t_j - 1),\ldots,x(t_j - 9)$ for the augmented state: $(101)^9$.
- Maximum number of training cycle lengths $y(tj)$: 3.

This generates an upper bound estimate for each week t of the state space of approximately $4.27x10^{20}$ possible states (i.e., $3x130x(101)^9$). Using complete enumeration to solve a two-year scheduling problem increases the size of the problem by approximately two orders of magnitude. On the other hand, dynamic programming may substantially reduce the amount of enumeration required to obtain an optimal solution. It does this by avoiding decision sequences that cannot possibly be optimal and by solving the problem one stage at a time. However, as Table 3 illustrates, the potential size of the augmented state space for the real-world problem, or a reduced problem, remains quite large. Other exact methods such as integer and mixed integer programming suffer from similar problems (see McGinnis, 1994). These obstacles motivated the development of efficient heuristics for solving the basic training scheduling problem.

4. HEURISTIC APPROACHES

The heuristic procedure presented here is applied in three phases. Phase I assumes an initial training requirement for each week t denoted by $\{r(1_1), r(2_1),\ldots,r(T_J)\}$. This is estimated by applying the historical distribution of trainee arrivals from Figure 3 to the recruiting target R_j for each year according to (6). Next, company strengths and training cycle lengths are initialized at $\overline{X}$ and $\underline{y}$, respectively. Then training companies are scheduled in each week to meet the training requirement. Idle training companies $I(t_j)$ are also computed for each week. If $I(t_j) < 0$ in any week t, then the current scenario is *infeasible*. Otherwise $I(t_j) \geq 0 \ \forall \ t,j$ and a feasible schedule must exist for the resources currently available.

Next, company strengths and training cycle lengths are reinitialized at $\underline{x}$ and $\overline{Y}$, respectively. This may cause a training company shortfall in some weeks where $I(t_j) < 0$. At this point of Phase I a *single-pass heuristic* (SPH) makes one forward pass through the planning horizon to correct training company shortfalls in each week where $I(t_j) < 0$. A policy iteration algorithm is applied a finite number of times to $x(t_j)$ and $y(t_j)$ until $I(t_j) \geq 0$. The single-pass heuristic repeats the policy iteration algorithm in each week of the planning horizon where $I(t_j) < 0$ thereby generating an initial feasible training schedule. The feasible scheduling policy for Phase I, signified by superscript 1, is specified by

$$\pi^1 = \begin{Bmatrix} x^1(1_1), x^1(2_1),\ldots,x^1(T_J-1); \\ y^1(1_1), y^1(2_1),\ldots,y^1(T_J-1) \end{Bmatrix}$$

Phase II considers options for changing the level of resources available to train soldiers. The

version of the model presented here only considers decisions that change the level of training companies available. However, the model is easily modified to consider other reusable resources. In practice, if the training scenario does not involve resource level changes, then Phase II may be omitted. The resource scheduling policy for Phase II is

$$\pi^2 = \begin{Bmatrix} x^2(1_1),\, x^2(2_1), \ldots, x^2(T_J - 1); \\ y^2(1_1),\, y^2(2_1), \ldots, y^2(T_J - 1); \\ d^2(1_1),\, d^2(2_1), \ldots, d^2(T_J - 1) \end{Bmatrix}$$

$d^2(t_j)$ represents the company deactivation decision in week t and superscript 2 denotes Phase II.

Experiments with the heuristic procedure have shown it is possible to improve resource schedules from Phase I. This observation led to the development of a *multi-pass heuristic* (MPH) that improves the resource scheduling policies obtained from the single-pass heuristic through a modified policy improvement step that further decreases company strengths.

Phase III uses the initial feasible schedule from Phase II as its starting point or π^1 from Phase I if Phase II is omitted. The multi-pass heuristic iteratively revises the company strength policy, week-by-week, working sequentially backward through the planning horizon. Within a week the modified policy improvement algorithm decreases company strength $x(t_j)$ by one step of size n at a time until no further improvement to the objective function is possible. The algorithm stops when either company strength reaches its lower bound $\underline{x}$ or $I(t_j) = 0$. The resource scheduling policy obtained at the completion of Phase III is

$$\tilde{\pi}^3 = \begin{Bmatrix} \tilde{x}^3(1_1),\, \tilde{x}^3(2_1), \ldots, \tilde{x}^3(T_J - 1); \\ y^2(1_1),\, y^2(2_1), \ldots, y^2(T_J - 1); \\ d^2(1_1),\, d^2(2_1), \ldots, d^2(T_J - 1) \end{Bmatrix}.$$

$\tilde{x}^3(t_j)$ denotes the "best" suboptimal company strength decision in week t given all previous decisions. $y^2(t_j)$ and $d^2(t_j)$ are the training cycle and training company deactivation decisions from Phase II.

5. DECISION SUPPORT SYSTEM (DSS)

The logistical complexities of the Army's initial entry training program create numerous practical decision problems. In many of these decision situations the best solutions are not obvious. For example, decision makers may be confronted by multiple competing objectives. In other situations decisions may depend upon a sequential decision process complicated by precedence constraints, or decisions may need to be evaluated over different planning horizons. The *Decision Support System for Army Basic Combat Training Resource Management per Year*, or *ARMY*, has been developed to study and methodically solve a broad range of basic training scheduling problems.

At the highest level of the Army, Department of the Army, force planners are responsible for properly manning the force. Force planners forecast recruitment objectives based upon personnel requirements derived from future military force structure. The determination of recruitment objectives generates annual recruiting targets that, in turn, drive training resource requirements. Recent events such as force structure downsizing and realignment and closure of military bases have complicated the force planning process. The *ARMY* system has been designed to analyze the impact of these events in terms of recruitment objectives and training program throughput. For example, demand for training company resources is determined by recruiting objectives that represents force structure personnel requirements. The *ARMY* system can also forecast training program throughput based on the level of training resources available to conduct training that, in part, reflects training base realignment and closure decisions. Both recruiting objectives and training companies are model parameters that are easily changed by the system user.

At TRADOC Headquarters, training program managers acquire and distribute training resources needed by training installations to accomplish Army training missions. The decision support system can help TRADOC training managers determine training loads for each training installation. The system estimates train-

ing costs for justifying training resource requirements and costs to Department of the Army. The system can also support studies and planning for special contingencies such as force mobilization and future changes to training programs due to force structure downsizing and base closures.

At the installation level, the decision support system can help training program managers resolve resource scheduling problems that affect training program execution. The *ARMY* system can develop precise training resource schedules that meet Department of the Army training requirements or determine that a feasible schedule does not exist for a given a training scenario. The DSS can also help installation training managers identify and resolve training resource shortfalls. Finally, the system enables training managers to generate efficient training resource schedules where training resources are highly utilized.

A future area of change for the basic training program will be the integration of computer and information technologies into the training environment. For example, the Army is presently developing a family of high technology, computer-based training facilities called the *Combined Arms Tactical Trainer* (see McGinnis and Phelan, 1996). These facilities will enable army units at the same or different locations to conduct realistic, interactive training in a virtual environment featuring a digitized battlefield. These systems are likely to be used in initial entry training as well. The Army is researching and developing new, innovative methods for teaching new soldiers the technical skills needed for operating and maintaining high-technology systems of the future. The curricula for future initial entry training programs will likely be substantially different from current curricula. In this area, the *ARMY* system may be used to study potential training program changes such as varying course lengths, classroom sizes, training company strengths, training program throughput, training resource requirements, and training program costs.

5.1 Decision Support System Development

Decision support system development was accomplished through four sequential, overlapping tasks.

1. Functional description of the system.
2. Preliminary design of system architecture and system modules.
3. Development of a system prototype.
4. Full system development.

In Task 1, the primary functions of the decision support system for supporting the scheduling decision process were identified. In Task 2, the system architecture was represented graphically through a set of interconnected modules. The modules embody the functional requirements of the system identified in Task 1. Four major issues were addressed during system architecture design.

1. System and module functionality.
2. System and module data exchange requirements.
3. Module procedures, logic, and rules for performing scheduling operations.
4. Data generation, storage, and retrieval requirements.

Figure 4 illustrates the DSS architecture and system modules.

In Task 3 module prototypes were developed and implemented in a computer spreadsheet called *LOTUS 1-2-3 for Windows*. The spreadsheet environment was ideal for streamlining the thousands of repetitive calculations required to generate a weekly training resource schedule. Scheduling procedures and rules for controlling the flow of data between modules and policy improvement routines were programmed using advanced macros. The spreadsheet software also provided built-in tools for statistical analysis of scheduling output. Once the system modules were performing as expected, information links were established between the modules for dynamic data exchange. Full system testing and system documentation in Task 4 concluded DSS development.

5.2 Description of the Decision Support System Modules

As shown in Figure 4, the centerpiece of the *ARMY* system is the dynamic mathematical model of the basic training program. The descriptive module names indicate the primary functionality of each module.

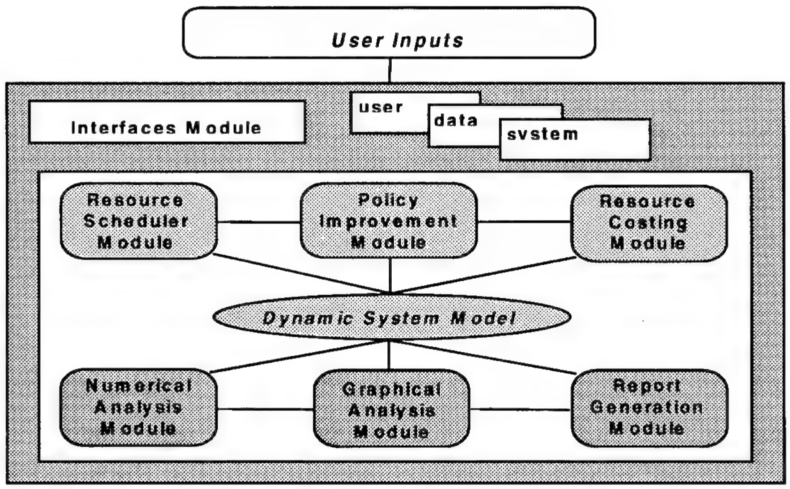

Figure 4: Decision support system architecture and system modules.

The *Resource Scheduler Module* first forecasts the number of trainees to arrive for training each week and then computes the number of training companies required to start training. Next, the module schedules basic training companies, or other resources, in each week of the planning horizon. Scheduling rules, constraints, and user-specified scheduling conditions describing the initial state of the training system are used within this module to schedule resources.

The *Policy Improvement Module* invokes the fully automated single-pass and multi-pass heuristic procedures to generate a good, feasible training resource schedule, if one exists. This eliminates the need for a system user to interact with the computer spreadsheet model as was the case when using the partially automated the manual *heuristics-used-in-practice* developed previously by McGinnis (1989).

The *Numerical Analysis Module* analyzes scheduling data, computes scheduling statistics, and summarizes scheduling information. The *Graphical Analysis Module* graphs scheduling information and statistics. The *Report Generation Module* produces numerical and graphical scheduling output tailored to the decision making needs of training managers. Section 6 gives examples of numerical and graphical output from the system. Output from an illustrative scheduling session with the *ARMY* system is given in Appendix C of McGinnis (1994).

The *Resource Costing Module* estimates various training program costs from the training resource schedules generated by the decision support scheduling system. Cost measures currently determined by the system include the following.

- Total annual training program costs. Total program costs are estimated for each year of a two-year scheduling horizon. Annual cost categories include fixed and variable costs by training resource.
- Annual training program cost variance. The annual training program cost variance represents cost differences between the two years of scheduling data generated by *ARMY*. The system expresses cost variances in constant dollars and also as percent differences in training costs from year-to-year. These differences reflect changes to training installations due to base closure and realignment or force structure changes measured as explained above. Comparisons of annual cost variances are made by total program cost, fixed and variable costs, and training resource costs.

1. In-processing activities	6. Laundry services
2. Basic training support	7. Food services
3. Supply operations	8. Personnel support
4. Maintenance of basic training equipment	9. Ammunition for weapons qualification
5. Transportation services	10. Utilities

Table 1. Cost items measured using the *ARMY* system.

- Average training program costs. The system computes various average cost estimates per training cycle. These include an average variable training cost for the entire training program as well as by training battalion, company, and trainee. The system also estimates average variable costs per training cycle by training company and resource. Finally, an average program cost per trainee and an average program cost per trainee by training resource are also estimated.

Cost data for this study was provided by the Directorate of Resource Management[1] (DRM) of Fort Benning, Georgia, based on a 1993 study of basic training resource utilization conducted at Fort Benning by the DRM. The *1993 Fort Benning Study* measured 14 training resource costs for a single basic training battalion. The training battalion consisted of a headquarters company and five basic training companies. Cost data were collected during six consecutive training cycles. Ten cost items from the Fort Benning study were used in this study to illustrate the costing methodology. The cost items are given in Table 1.

6. RESULTS

Section 6.1 compares scheduling results from the heuristic methods for 12 realistic test scenarios. The number of training companies available in each scenario reflect potential base realignment and closure decisions that differentiate the test scenarios. The results are compared using three performance measures: processing time to generate a training schedule, training quality, and training resource utilization. See McGinnis (1994) for a comparison of scheduling results based on basic training costs. Three sets of scheduling results are presented for each test scenario and performance measure. One set of results was generated using a semi-automated Heuristics-Used-In-Practice (HUIP) procedure developed previously by McGinnis (1989). The other two sets came from the Single-Pass Heuristic (SPH) alone and the combined Single-Pass and Multi-Pass Heuristic[2] (SPH-MPH).

Section 6.2 compares optimal scheduling solutions from dynamic programming with results from the combined Single-Pass and Multi-Pass Heuristic procedure. As discussed in Section 3, the size of the basic training scheduling problem precluded implementation of exact solution methods for the real-world problem. Nevertheless, we implemented dynamic programming for a much simplified version of the basic training problem. These results serve as a yardstick for measuring the quality of feasible schedules obtained using the heuristic.

Test Scenarios

The test scenarios for this study reflect 1990 training base structure and approximate recruiting targets for 1989 and 1990. Table 2 gives the number of training companies deactivated by year and scenario (see D_{1989} and D_{1990}). The annual recruiting targets for the scheduling horizon were R_{1989} = 136,000 and R_{1990} = 128,000. The scheduling system was initialized with 130 training companies at the beginning of the first year of the planning horizon and 10-week training cycle lengths throughout. Training company strengths for all test scenarios were initialized at 200 trainees per company for the HUIP procedure and 150 trainees for the SPH and SPH-MPH procedures except for Scenarios 10 and 12. In these two cases, the company strengths of all three scheduling methods were initialized at 150 and 200 trainees per company, respectively.

6.1 Heuristic Scheduling Results

This section evaluates training resource schedules from the heuristic scheduling procedures using three performance measures.

			HUIP			SPH			SPH-MPH		
Scen	D_{1989}	D_{1990}	Time	Ratio	Idle Co	Time	Ratio	Idle Co	Time	Ratio	Idle Co
1	5	0	2:57	.478	34	:53	.581	8	1:29	.582	8
2	10	0	3:53	.476	30	:58	.568	6	1:36	.569	6
3	15	0	5:53	.473	25	1:01	.554	5	1:39	.556	5
4	20	0	9:52	.469	21	1:06	.540	4	1:53	.542	4
5	0	5	3:03	.480	36	:51	.585	9	1:27	.587	9
6	0	10	3:21	.478	34	:53	.577	8	1:32	.578	8
7	0	20	3:23	.471	31	:57	.559	7	1:41	.561	7
8	0	30	9:57	.464	28	1:05	.542	7	1:53	.544	7
9	5	5	5:09	.477	32	:56	.572	7	1:38	.574	7
10	10	10	10:27	.515	8	1:05	.549	6	1:44	.551	6
11	15	15	11:48	.465	20	1:13	.527	4	2:08	.529	4
12	10	10	4:51	.472	25	:41	.470	22	12:35	.549	7

Table 2. Numerical scheduling results for the heuristic methods.

1. Schedule processing time. The time to obtain a feasible schedule.

2. Training quality. Measured in terms of an instructor-to-trainee ratio $\sum_{t_j=1_1}^{T_J} \frac{1}{x(t_j)}$

where we assume one instructor per training company for simplicity. The *utopian* value for training quality based on company strengths of 150 trainees and a two-year scheduling horizon is 0.64, so named because it will not usually be obtainable under normal circumstances.

3. Training resource utilization. Measured by the average number of idle training companies per week . $\frac{1}{T_J} \sum_{t_j=1_1}^{T_J} I(t_j)$

Scheduling Results

The first author conducted all computational tests on a 486/66 desktop computer. Table 2 gives numerical results by training base scenario (*Scen*) and scheduling method (*HUIP, SPH, SPH-MPH*) for the performance measures cited above. The test cases were limited to scenarios where the scheduling methods generated feasible schedules. This established a common basis for comparing the scheduling results by heuristic method and performance measure. The entries in each cell of columns D_{1989} and D_{1990} of Table 2 indicate the number of training companies "deactivated" each year. Processing *Time* reflects the time in minutes and seconds to generate a training resource schedule. *Ratio* gives the instructor-to-trainee ratio values summed over the two-year planning horizon of T_J weeks. *Idle Co* represents the number of idle training companies averaged over the planning horizon and rounded to the nearest whole training company.

Heuristic Scheduling Processing Time

Figure 5 graphs training schedule processing times for the heuristic procedures. For Scenarios 1 through 11, the SPH procedure generated schedules 6.4 times faster than the HUIP procedure and 1.7 times faster than the combined SPH-MPH procedure. The combined SPH-MPH procedure was 3.7 times faster than the HUIP procedure on average. In Scenario 12, company strengths were initialized at 200 trainees modeling the initial condition of the heuristics-used-in-practice. Although this initial condition dramatically reduced schedule generation time using the SPH procedure, it also led to significantly poorer SPH results in terms of training quality and training resource utilization (see Table 2). It also substantially increased the processing times for the combined SPH-MPH procedure since many more iterations of the policy improvement algorithm were necessary to improve the company strength policy.

Instructor-to-Trainee Ratio

As discussed in Section 2, the instructor-to-trainee ratio reflects company strength decisions

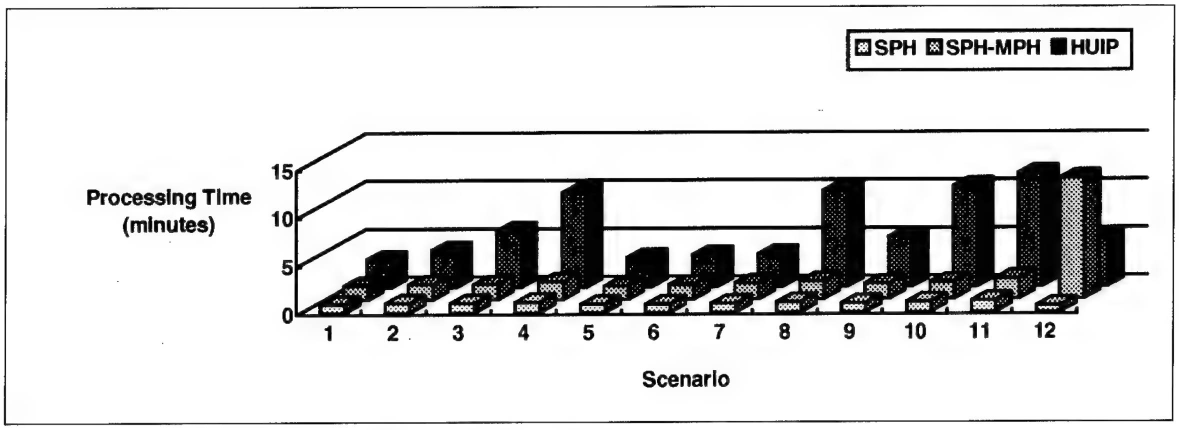

Figure 5: Heuristic processing times.

in each week and also serves as a performance measure of training "quality." Therefore, the problem is suboptimized by making company strengths as small as possible. Figure 6 gives the ratio values for the 12 test scenarios. On average, the quality of the SPH and SPH-MPH schedules for Scenarios 1 through 11 were 18.7% and 19% better than the HUIP schedules, respectively.

Although the SPH-MPH improved SPH solutions by only 0.3%, this modest improvement in the "quality" of training may be significant in terms of total annual training program throughput. In Scenario 12, initializing company strengths at 200 imposes a "penalty" on the quality of SPH scheduling solutions. However, Scenario 12 also illustrates the how the MPH procedure methodically tightens company strengths in each week of the planning horizon to improve the quality measure. The MPH made a substantial 16.8% improvement in the SPH-MPH objective function value: 0.470 for the SPH versus 0.549 for the SPH-MPH.

Figure 7 compares the scheduling results as a percentage of the *utopian* value where 100% corresponds to an instructor-to-trainee ratio of 0.64.

Figure 7 also illustrates the effects of the training company initial condition on the "quality" of training schedules. Excluding Scenarios 10 and 12, the HUIP, SPH, and MPH methods generated solutions that were, on average, approximately 74%, 87.5%, and 87.8% of the *utopian* training "quality" value.

Average Number of Idle Training Companies

The average number of idle training companies for each training resource schedule may be interpreted as a measure of training company

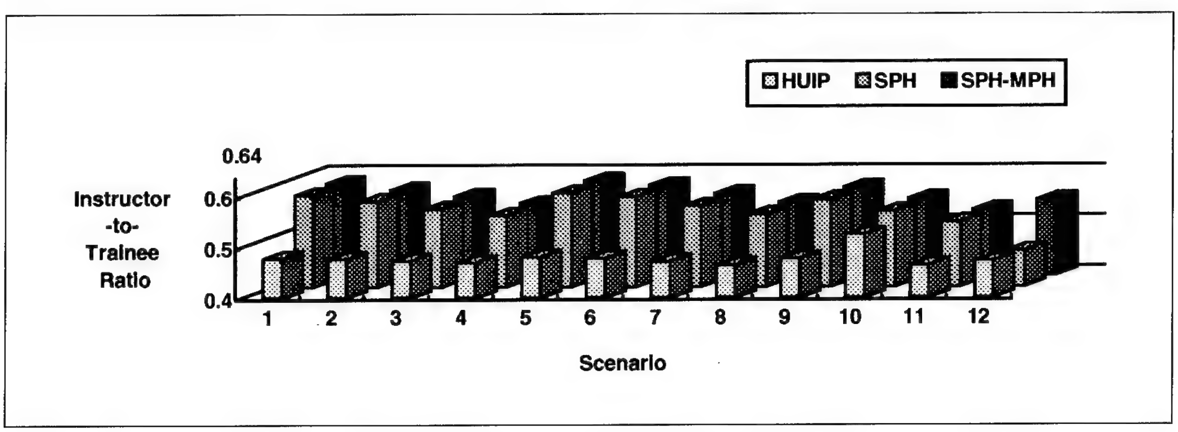

Figure 6: Instructor-to-Trainee Ratios.

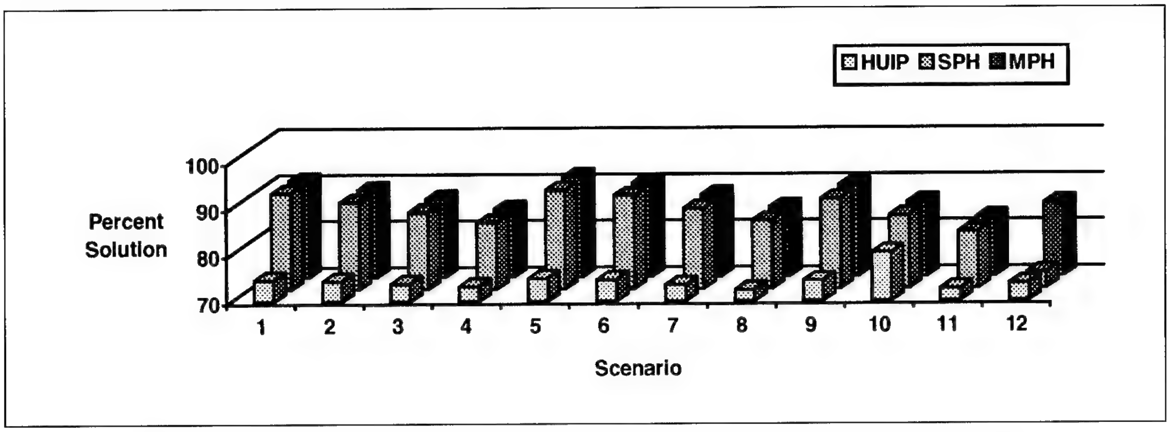

Figure 7. Heuristic solutions measured as a percentage of the *utopian* value.

utilization. For this measure, the objective is to generate a training schedule that "minimizes" idle training companies. Coincidentally, this is achieved by "maximizing" the instructor-to-trainee ratio in each week. As explained above, this is accomplished by making company strengths as small as possible. As shown in Figure 8, the average number of idle training companies is also highly dependent upon the number of training companies available for training. Obviously, when fewer companies are available to train soldiers, the remaining companies are used more often leading to a higher training company utilization and fewer idle companies in each period.

Figure 8 also reveals that for all scenarios except 12, idle training companies determined from the SPH and SPH-MPH procedures are exactly equal. The company strength policy improvement algorithm of Phase I increases $x(t_j)$ in weeks where $I(t_j) < 0$ by one step of size n at a time. The instant the constraint becomes tight, that is $I(t_j) = 0$, the algorithm stops. Therefore, although the MPH procedure may improve the company strength policy in each week, it cannot further reduce idle training companies. However, when training company strengths are initialized at values greater than the lower bound, such as at 200 trainees in Scenario 12, then there may be numerous weeks where $I(t_j) >$ 0. Refinement of company strengths in these weeks by the MPH procedure may reduce idle training companies as well. The SPH-MPH procedure generated schedules that were 3.8 times more efficient at utilizing resources, on average, than the HUIP procedure (excluding Scenario 12).

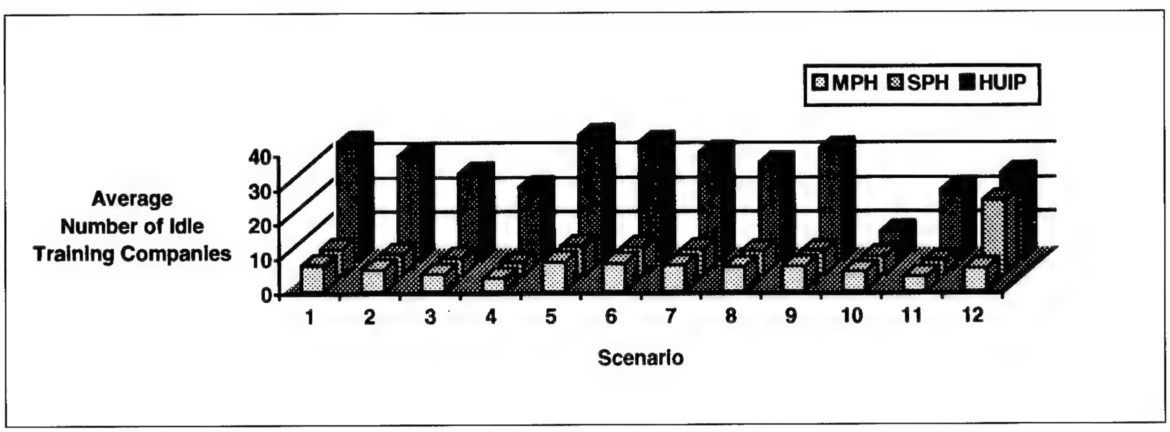

Figure 8. Idle training companies

6.2 Comparison of Optimal and Heuristic Scheduling Results

As shown in Section 3, the size of the real-world basic training problem makes it impractical to use exact solution procedures for solving the problem. However, we implemented dynamic programming for a simplified version of the problem in order to compare heuristic and optimal scheduling results. This required the following modifications to the problem.

- Training cycle length decisions were removed by fixing cycle lengths at $y(t_j) = 2$. This reduced the information time lag from 9 weeks to 1 week.

- The allowable number of idle training companies $I(t_j)$ was reduced from 130 to 20 based on experimental results from testing the automated heuristic.

- The allowable company strength decisions $x(t_j)$ were reduced from 101 reflecting a step size of 1 to 21 for a step size of 5.

These modifications reduced the size of one stage of the problem from approximately $4.27x10^{20}$ possible states to 9,261 states (i.e., $21x21x21$). The planning horizon was reduced from 96 to 48 weeks. This established a new *utopian* value of 0.32 for the training "quality" performance measure. Although shortening the planning horizon does not affect the size of the state space, it does reduce computer memory required to generate and store training schedules as well as processing time.

Implementation of the DP Algorithm

The state transition equation for a one-week time lag problem and a one-year planning horizon is given by

$$I(t+1) = I(t) - \frac{r(t)}{x(t)} + \frac{r(t-1)}{x(t-1)}$$

Company strengths $x(t)$ remain restricted to the allowable set of integer-valued decisions $X[t, I(t)] \subset \Omega$ but modified as explained above. Despite the seemingly trivial nature of this problem, implementation of the dynamic programming algorithm for the one-period time lag problem represents an important first step towards solving larger, more complex problems. The steps to implement the algorithm for larger problems remain essentially the same for the one-week time lag problem with two obvious exceptions. First, the size of state space for the real-world problem increases exponentially with state space augmentation. Second, the complexity of implementing dynamic programming to solve the real-world problem increases substantially as additional variables are incorporated into the state transition equation to deal with information time lag. Table 3 gives state space increases for information time lags based on a company strength step of five.

Time Lag (l)	$x(t-l)$	$I(t)$	$x(t)$	State Space
1	21	21	21	9,261
2	21^2	24	21	222,264
3	21^3	27	21	$5.2x10^6$
4	21^4	30	21	$1.2x10^8$
5	21^5	33	21	$2.8x10^9$
6	21^6	36	21	$6.5x10^{10}$
7	21^7	39	21	$1.5x10^{12}$
8	21^8	42	21	$3.3x10^{13}$
9	21^9	45	21	$7.5x10^{14}$
10	21^{10}	48	21	$1.7x10^{16}$

Table 3. State space sizes for each 1-period increase in information time lag.

Dynamic Programming Versus Heuristic Results

The allowable initial states for the dynamic programming solution to the one-period time lag problem generated a total of 441 scheduling solutions. Table 4 compares dynamic programming and heuristic scheduling results for 12 representative test scenarios. The *SPH-MPH* and *DP* columns give the objective function values for these scheduling procedures, respectively. *% Solution* expresses the heuristic solutions as a percentage of the dynamic programming solutions for each test scenario.

The results illustrate the influence of initial scheduling conditions for company strengths and idle training companies on the objective function values for the heuristic procedure. On average, the SPH-MPH procedure generated scheduling solutions that were approximately 91% of the optimal solution for the one-period lag problem. Figure 9 illustrates how changes in the initial values of $x(t)$ for each value of $I(t)$ dramatically change the "quality" of heuristic solutions relative to optimal solutions.

7. CONCLUSIONS

This paper solves a complex scheduling problem of practical interest to the United States Army. Specifically, the problem is one of scheduling reusable training resources for the Army's Basic Combat Training program over a finite planning horizon. Notable dynamic characteristics of the basic training system model include the following.

1. Seasonal arrival of trainees that represent demand for training resources.
2. Varying training company strength.
3. Varying training cycle length.

Test Scenario	$I(t)$	$x(t\text{-}1)$	SPH-MPH	DP	% Solution
1	0	150	.3047	.3121	98 %
2	0	200	.2831	.3103	91 %
3	0	250	.2652	.3027	88 %
4	1	150	.3071	.3121	98 %
5	1	200	.2981	.3113	93 %
6	1	250	.2721	.3063	89 %
7	2	150	.3091	.3121	99 %
8	2	200	.2942	.3119	94 %
9	2	250	.2773	.3084	90 %
10	3	150	.3108	.3121	99 %
11	3	200	.2983	.3121	96 %
12	3	250	.2831	.3103	91 %

Table 4. Scheduling results for the 1-period time lag problem.

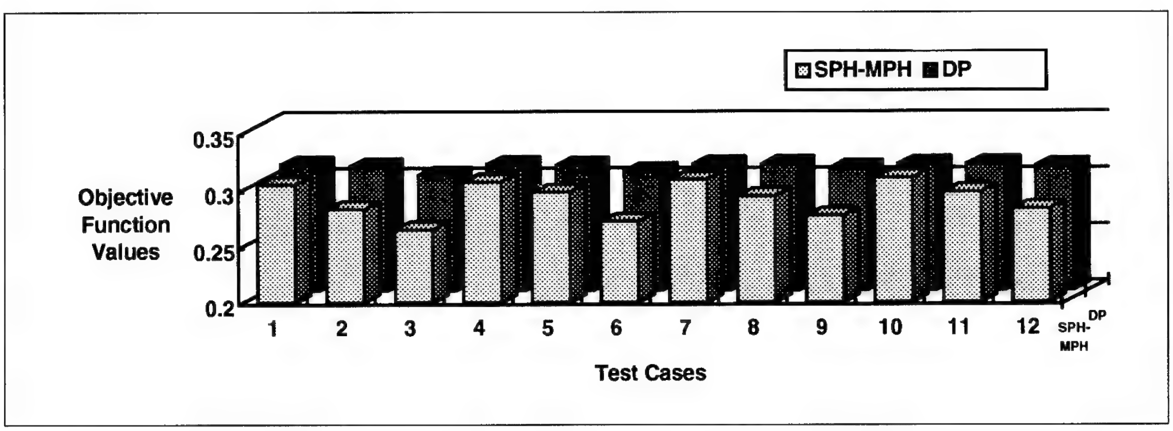

Figure 9. Comparison of heuristic and optimal solutions for a 1-period time lag problem.

In real-world basic training, trainees are given a window of several weeks to report for training. This makes trainee arrivals a random process that, in turn, makes the demand for training resources stochastic. However, we estimate trainee arrivals ahead of time for each week of the planning horizon which makes our problem deterministic.

7.1 Benefits of the Decision Support System

The decision support system permits system users at different levels of decision making to study a broad range of practical problems. For example, at Department of the Army, the system can be used to examine issues such as the impact of consolidating, closing, or reducing the size of training installations on training program throughput. Alternatively, the system can estimate training program throughput for meeting future Army force structure requirements in terms of recruitment objectives. It can also be used to evaluate the feasibility of scenarios for expanding the training base structure during mobilization. The *ARMY* system generates potentially useful output such as the average number of idle training companies that helps determine whether the training base is either over or under structured relative to projected recruitment targets. Finally, the system can support analysis of alternatives for consolidating military training programs into "joint" training centers where military personnel from all services may be trained at the same installation.

At Training and Doctrine Command (TRADOC) Headquarters the system can help training program managers evaluate the economic impact of different resource utilization policies if measured in terms of training capacity and training program throughput. This is done by varying system model parameters such as recruiting levels, training program duration (course lengths), or the level of training resources available.

Finally, at the training installation level, *ARMY* can generate precise training resource schedules, forecast training resource requirements, and estimate training program costs. *ARMY* provides installation training program managers with a fully automated, computer-based scheduling system for generating "good" weekly schedules of reusable training resources in reasonable time. These schedules have high practical value as a preliminary step to developing executable training resource schedules for day-to-day operations.

The system employs practical scheduling performance measures to provide decision makers with a rational basis for selecting the "best" alternative from competing, feasible ones. The performance measures include training program quality as measured by the instructor-to-trainee ratio, training program throughput, estimates of resources required to meet training objectives, and training program costs. Despite the current Army basic training orientation of the system, it can be adapted to other Army training programs and to training programs of other the other military services as well.

7.2 Summary of Results

Experimental results from testing real-world scenarios suggest that the SPH-MPH procedure can determine solutions that are within 10% to 15% of the "best" (utopian) solution based on (7). These results were obtained in reasonable time on a 486 microcomputer. The combination of good, timely results establishes the potential future value of the heuristic to basic training management and related decision processes. For the one-period time lag problem the heuristic achieved results that were 91% of optimal for a small but representative set of test cases. However, it is expected that the scheduling performance of the heuristic will decline relative to exact solution methods as the size of the problem increases.

Although substantial computational effort is necessary to generate dynamic programming solutions, the results are very valuable. First, the quality of the dynamic programming solutions is superior to the heuristic results. Second, the dynamic programming algorithm generates the optimal training schedule, the optimal objective function value for each schedule, and the scheduling decisions to make in each period for every initial state. Once obtained, the results are easily stored and retrieved using a simple table "look up" procedure. This approach is ideal for quickly generating training schedules but impractical for "what-if" analysis.

NOTES

[1] Major Scott Manderville, then of the Directorate of Operations and Training (DOT), Fort Benning, Georgia, provided cost data for the study.

[2] SPH-MPH notation denotes that the SPH procedure generates an initial feasible training resource schedule that serves as the starting point for the MPH procedure.

ACKNOWLEDGMENTS

We thank Ferenc Szidarovszky of the University of Arizona for his insightful review of the mathematical formulation of the problem. The reviewer comments and suggestions for improving the paper were particularly helpful and appreciated. The first author thanks Lieutenant General Theodore Stroup, Colonel Dave Hardin, and Mr. Rich Wagner, then of TRADOC Headquarters, for their support as the original sponsors of this work. Finally, we thank Colonel James Kays, Head, Department of Systems Engineering, West Point, New York, for his advice and support.

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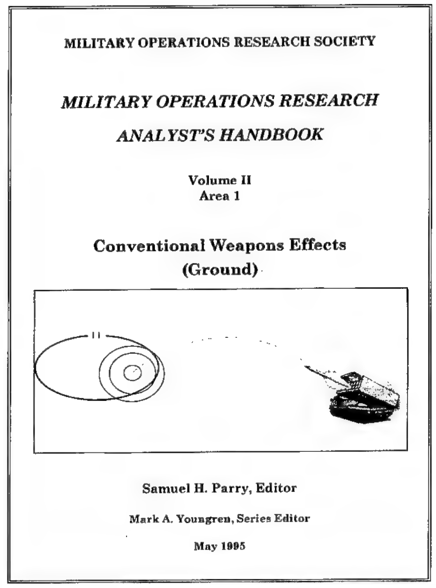

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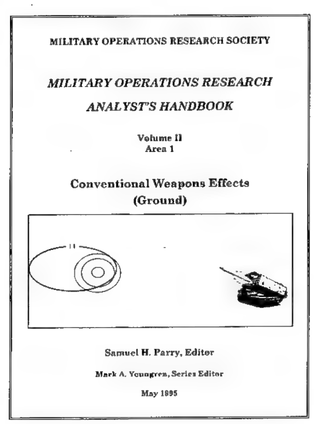

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MILITARY OPERATIONS RESEARCH: RESPONDING TO CHANGE

by LtCol Paul F. Auclair
Edward F. Mykytka
and Dr. Gregory S. Parnell

Paul F. Auclair is the Head of the Department of Operational Sciences in the Graduate School of Engineering at the Air Force Institute of Technology. His career experience includes tours in ICBM operations, ICBM performance assessment, and strategic missile operations analysis at the Pentagon. He received a B.S. from the Air Force Academy in 1976, an M.S. in Operations Research from the Air Force Institute of Technology in 1982, and a Ph.D. in Industrial and Systems Engineering from the Georgia Institute of Technology in 1993. He research interests include Strategic Missile Operations Analysis, Operational Test and Evaluation, CEP Estimation Techniques, Design of Experiments, and Response Surface Methodology.

Edward F. Mykytka is an Associate Professor of Operations Research and serves as the Deputy Head of the Department of Operational Sciences in the Graduate School of Engineering at the Air Force Institute of Technology. He received a B.S. in Mathematics from the University of Dayton in 1976; an M.S. in Industrial Engineering from the University of Iowa in 1978; and a Ph.D. in Systems Engineering from the University of Arizona in 1983. His research interests include discrete-event system simulation (particularly random variate modeling and generation) , quality improvement and statistcal process control, stochastic modeling , and applied statistics.

Greg Parnell teaches decision and risk analysis, mathematical programming, and probability at Virginia Commonwealth University. His research and consulting involves decision & risk analysis of military and environmental programs. He is a retired AF Colonel and former President of MORS. He runs every day the sun comes up.

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by Dr. William F. Bowlin

William F. (Bud) Bowlin teaches cost accounting and government and not-for-profit accounting at the University of Northern Iowa. His research focuses on performance measurement and evaluation. He has previously published in *Annuals of Operations Research, Journal of the Operational Research Society, Omega, Journal of Cost Analysis,* and other journals. Bud is a retired AF Lt. Colonel and travelled the highways between the Pentagon and Herndon, Va for several years.

BOMB DAMAGE ASSESSMENT AND SORTIE REQUIREMENTS

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Dennis Evans is an operations research analyst at the Office of the Director for Program Analysis and Evaluation, Office of the Secretary of Defense. His specialities are bombers and bomber weapons, and he is involved in modeling air warfare.

MODELING COST AND SCHEDULE UNCERTAINTIES—A WORK BREAKDOWN STRUCTURE PERSPECTIVE

by Paul R. Garvey

Mr. Garvey is Principal Scientist in the Economic and Decision Analysis Center at the MITRE Corporation. Mr. Garvey leads MITRE's risk management and analysis work program. His technical articles on this topic have appeared in books published by IEEE Computer Society Press, Springer-

Verlag's economics and mathematical systems series, and in professional journals such as Military operations Research (MOR) and the Journal of Cost Analysis. He is presently completing a textbook "Probability Methods for Cost Uncertainty Analysis—A Systems Engineering Perspective." Recently, Mr. Garvey was among 100 NATO scientists to brief its Operations Analysis Group on quantifying uncertainty in defense decision-making. He is a four-time best paper award recipient from the Department of Defense at its annual cost analysis symposium.

DYNAMICAL INSTABILITY IN COMBAT MODELS: COMPUTER ARITHMETIC AND MATHEMATICAL MODELS OF ATTRITION AND REINFORCEMENT

by Dr. Julian I. Palmore

Julian Palmore teaches differential equations and dynamical systems theory at the University of Illinois, Urbana. He is on the faculties of Mathematics and Computational Science and Engineering, an interdisciplinary program. He does research on computational chaos and its effects on computer simulations. He is Editor of *PHALANX* and of selected MORS Proceedings.

MILITARY TRAINING RESOURCE SCHEDULING: SYSTEM MODEL, OPTIMAL AND HEURISTIC DECISION PROCESSES

by LTC Mike McGinnis,
Professor Emmanuel Fernandez-Gaucherand
and Dr. Pitu B. Mirchandani

LTC Mike McGinnis, an Associate Professor at the U.S. Military Academy, is currently the Director of the Academy's Operations Research Center. He received a B.S. degree from USMA, M.S. degrees in Applied Mathematics and Operations Research from Rensselear Polytechnic Institute, Troy, New York, and a Ph.D. from the University of Arizona, Tucson. LTC McGinnis attended the Naval War College on full athletic scholarship where he graduated with Distinction and received an M.A. in National Security and Strategic Studies. His applied interests are in resource scheduling, military personnel policy problems, and command and control.

Professor Emmanuel Fernandez-Gaucherand has advanced degrees in electrical engineering and applied mathematics. He obtained a Ph.D. in Electrical and Computer Engineering from the University of Texas at Austin in August 1991. He is currently an Assistant Professor in the Systems and Industrial Engineering Department at The University of Arizona. Dr. Fernandez' research interests are in stochastic models, dynamic programming, and optimal decision and control processes. His applied interests are in logistics, management, telecommunication systems, and algorithm and software development with over 44 archival publications. As an expensive hobby, he is trying to learn how to beat the stock market.

Dr. Pitu B. Mirchandani is a Professor and Department Head of Systems & Industrial Engineering at the University of Arizona. He has B.S. and M.S. degrees from UCLA in General Engineering and Control Systems, respectively, a S.M. degree from MIT in Aeronautics and Astronautics, and a Sc.D. degree, also from MIT, in Operations Research. Dr. Mirchandani's areas of technical expertise include systems modeling and design, planning, real-time scheduling and control, logistics, optimization, and decision support systems, with applications in transportation and manufacturing. He has written over 60 publications and two books including Location on Networks; Theory and Algorithms (MIT Press, 1979) and co-edited Discrete Location Theory (John-Wiley, 1990).

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